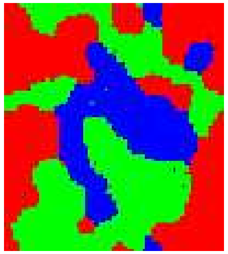
LGT results on heavy quark potentials and heavy quark spectral functions

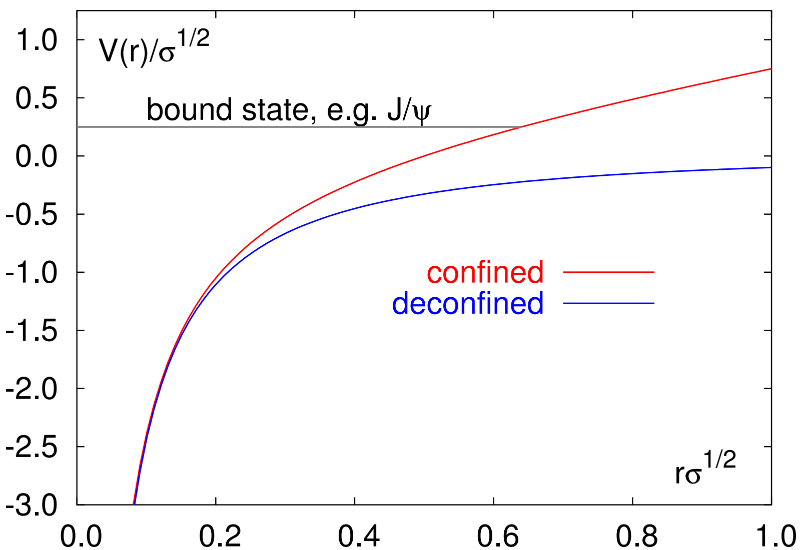
- Introductory remarks
  - Deconfinement, screening and asymptotic freedom
- Potential models for heavy quarkonium
  - length scales from (free) energies, Schrödinger Eq.
- Hadron correlation functions and spectral functions
  - charmonium and bottomonium at high temperature
- Closing remarks
  - Sequential suppression or statistical hadronization in heavy ion collisions



Deconfinement $\Rightarrow$ screening $\Rightarrow$ quarkonium suppression

The Matsui-Satz argument:

- deconfinement $\Rightarrow$ screening
  $\Rightarrow$ no heavy quark bound states in a QGP

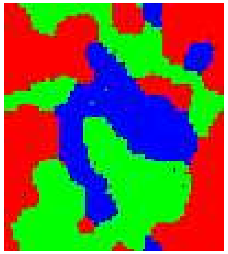


$V_{\bar{q}q}(r,T) \rightarrow \infty$ confinement

$V_{\bar{q}q}(r,T) < \infty$ deconfinement

- heavy $q\bar{q}$-pairs are rare states in a QGP
  $\Rightarrow$ dissolved pairs never recombine

Deconfinement ⇒ screening ⇒ quarkonium suppression



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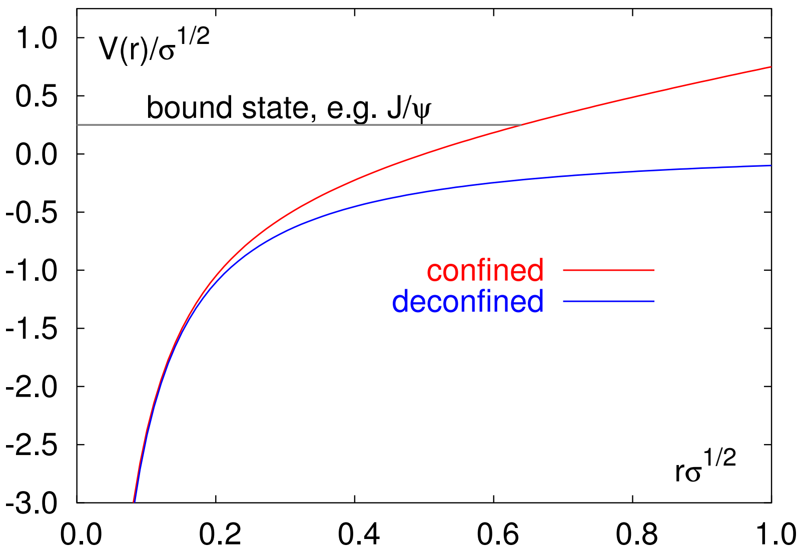

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J/ψ suppression

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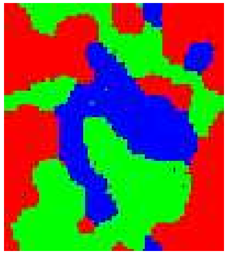
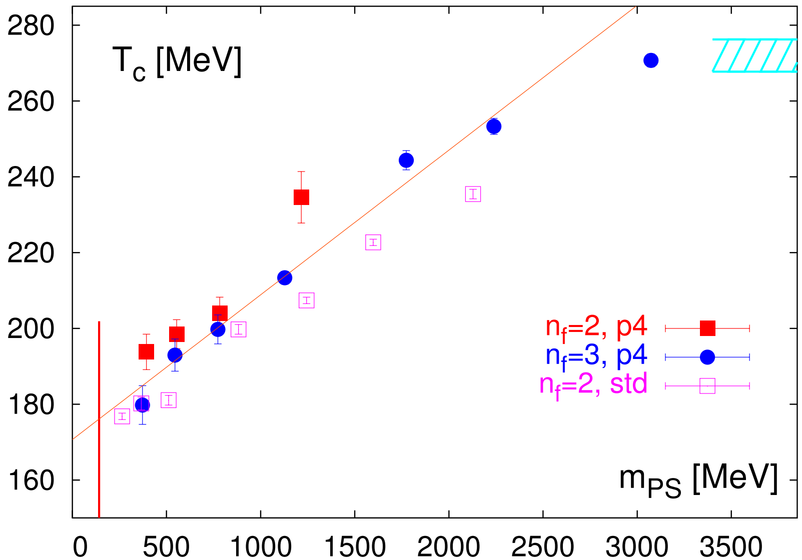


Critical temperature and hadronic resonances



⇐ understood in terms of an exponentially rising energy spectrum for string fluctuations:

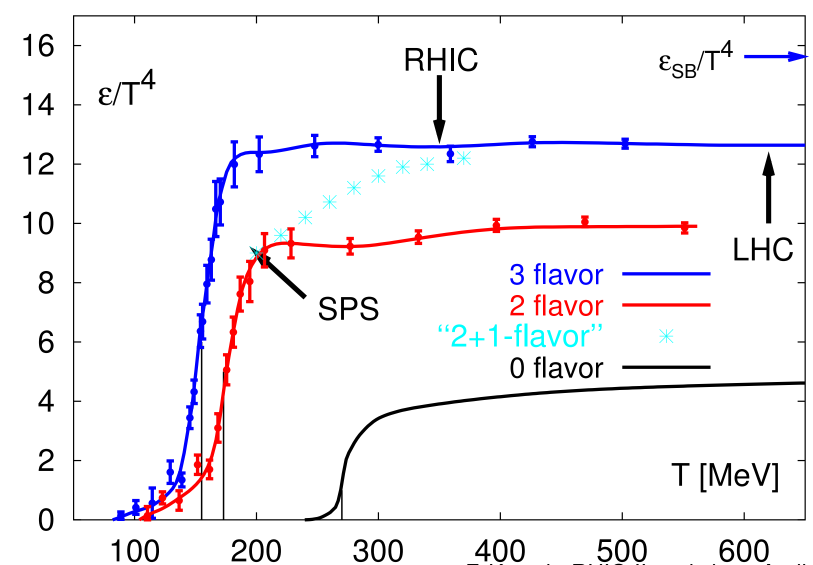
$$\frac{T_c}{\sqrt{\sigma}} \simeq \sqrt{\frac{3}{(d-2)\pi}}$$

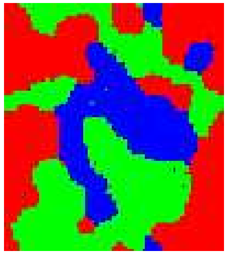
⇒ resonance gas

$m_{PS} \simeq 140\ MeV: \quad T_c \simeq 170\ MeV$

$m_{GB} \simeq 1.5\ GeV: \quad T_c \simeq 270\ MeV$
$(m_{PS} = \infty)$

energy density for 0, 2 and 3-flavor QCD





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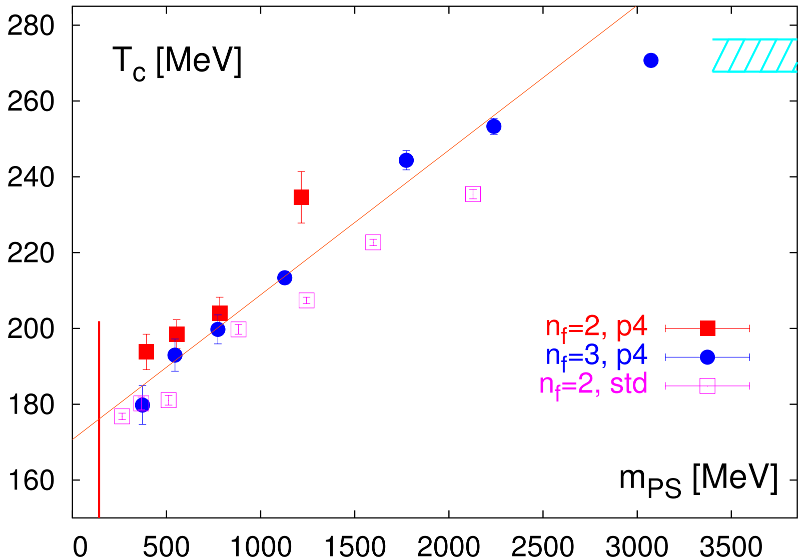

$\Leftarrow$ understood in terms of an exponentially rising energy spectrum for string fluctuations:

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$\Rightarrow$ resonance gas

$$m_{PS} \simeq 140\ MeV : \quad T_c \simeq 170\ MeV$$

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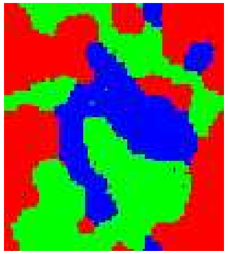
$(m_{PS} = \infty)$

$$n_f = 2 : \quad \epsilon_c \simeq (6 \pm 2)\ T_c^4$$
$$\simeq (0.3 - 1.3)\ GeV/fm^3$$
$$n_f = 0 : \quad \epsilon_c \simeq (0.5 - 1)\ T_c^4$$
$$\simeq (0.3 - 0.7)\ GeV/fm^3$$

change in ϵ_c/T_c^4 compensated by shift in T_c
transition sets in at similar energy densities
$\Rightarrow$ percolation



Confinement and deconfinement

confinement

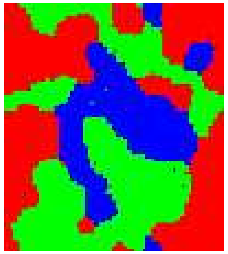
- stick together, find a comfortable separation
- controlled by confinement potential

$$V(r) = -\frac{4}{3}\frac{\alpha(r)}{r} + \sigma r$$

deconfinement

- free floating in the croud
- average distance always smaller than r_{af}:

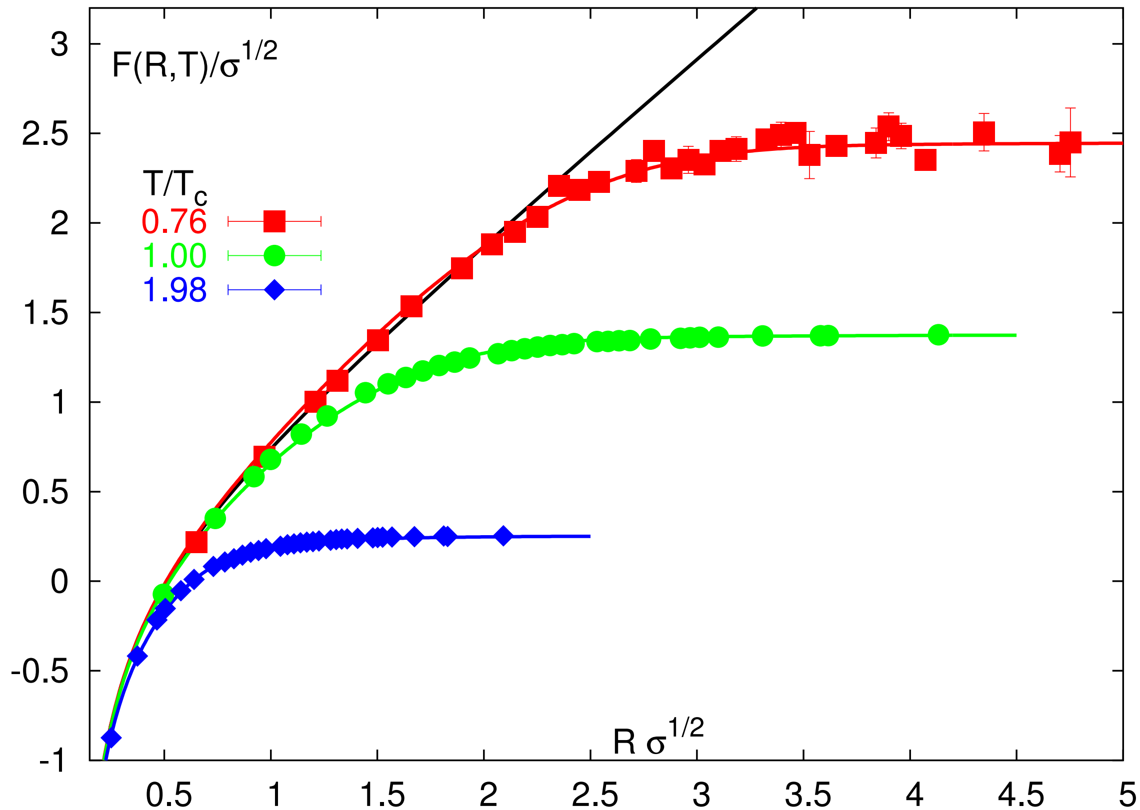
$$r_{af} = \sqrt{\frac{4}{3}\frac{\alpha(r)}{\sigma}} \simeq 0.25\,\mathrm{fm}$$

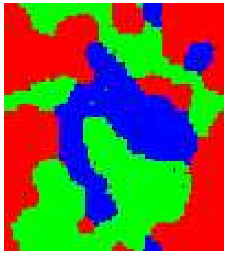


Length scales from heavy quark free energies

pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, hep-lat/0406036
2-flavor QCD: O.Kaczmarek, F. Zantow, hep-lat/0503017

- exponential damping at large distances defines $r_D \equiv 1/m_D$:
- $F_\infty \equiv V(r)$ gives an estimate for distance scale r_{med} beyond which medium effects are large





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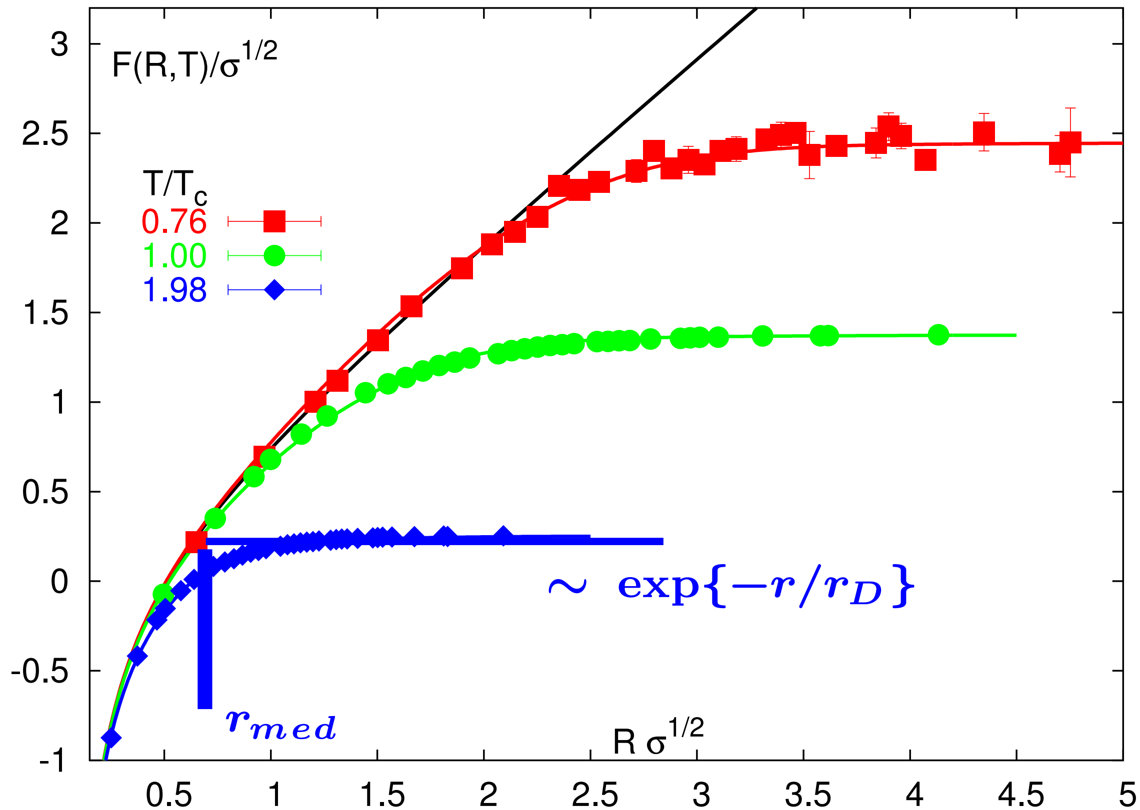

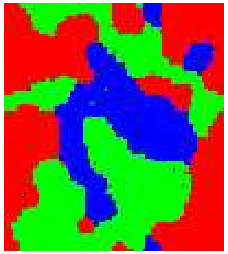

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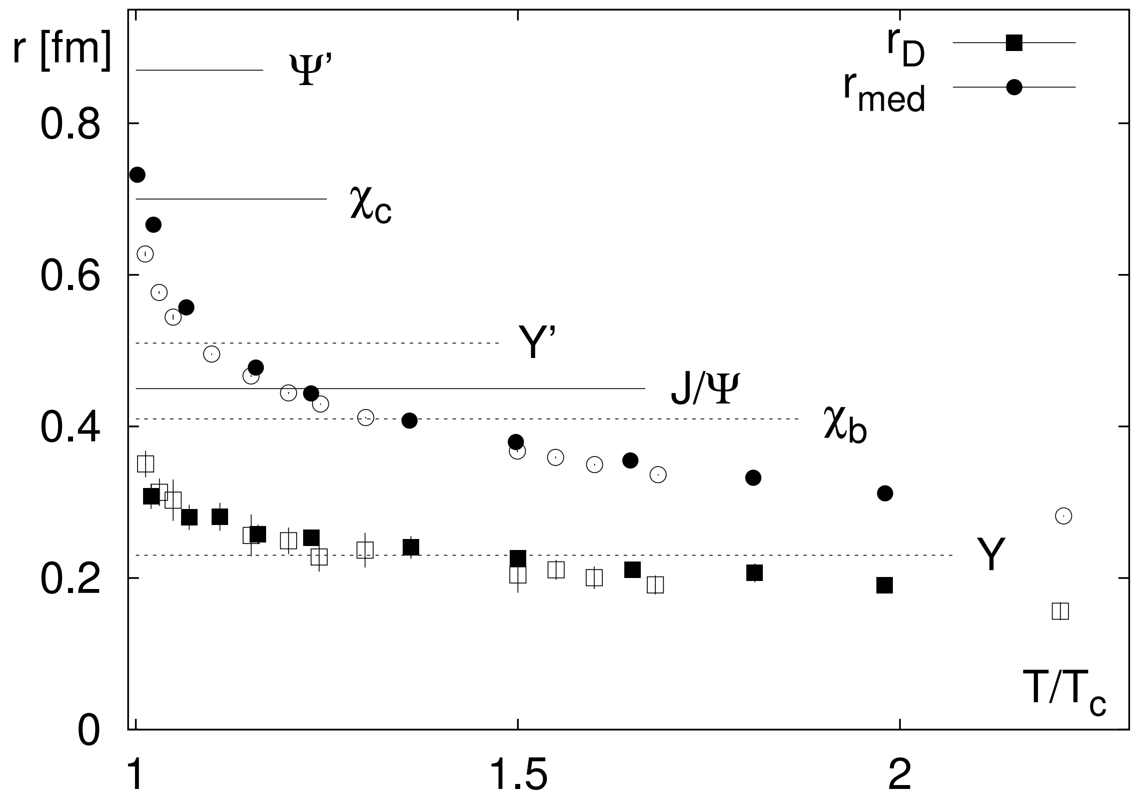

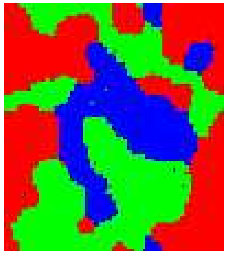

Singlet free energy and asymptotic freedom

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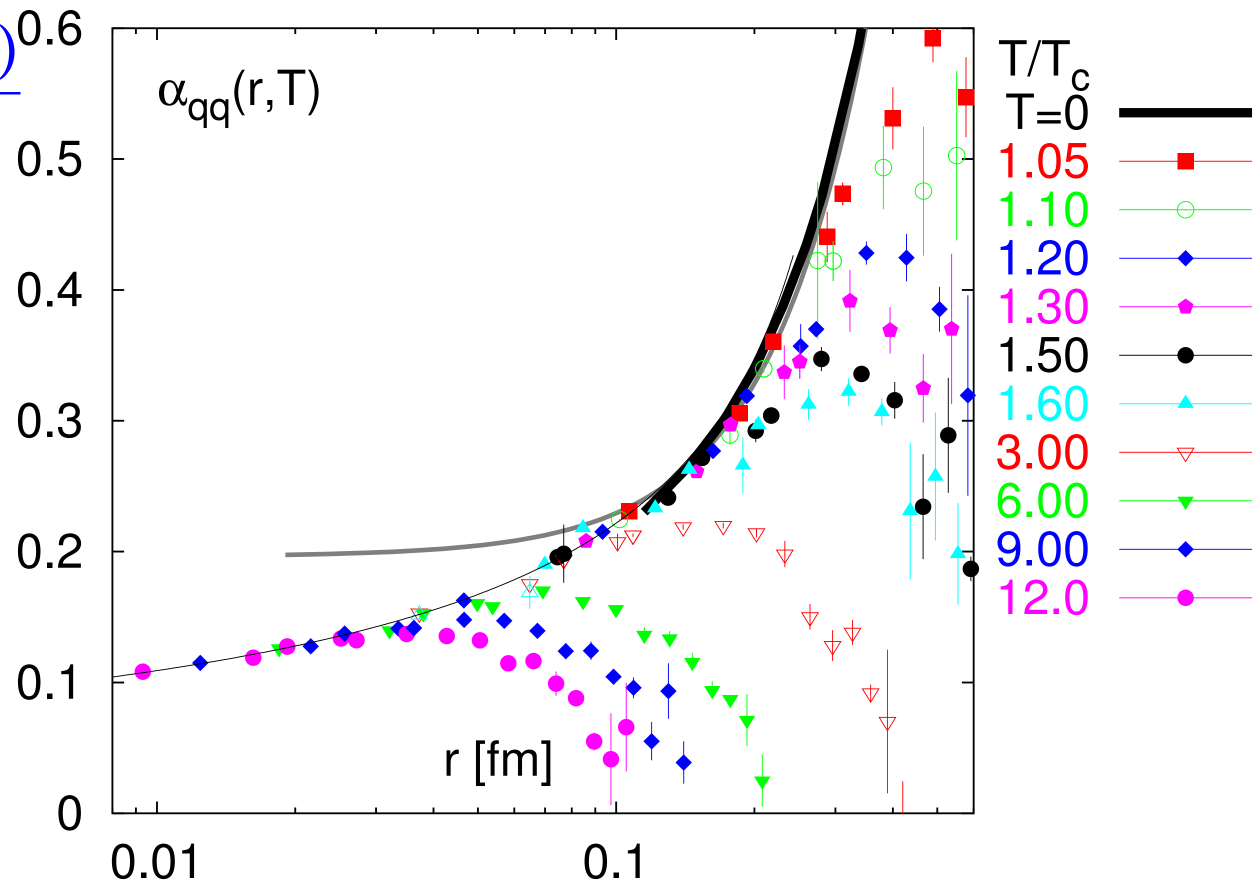
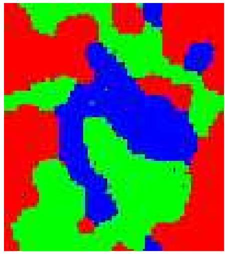
- singlet free energy defines a running coupling:

$$\alpha_{\rm eff} = \frac{3r^2}{4}\,\frac{{\rm d}F_1(r,T)}{{\rm d}r}$$





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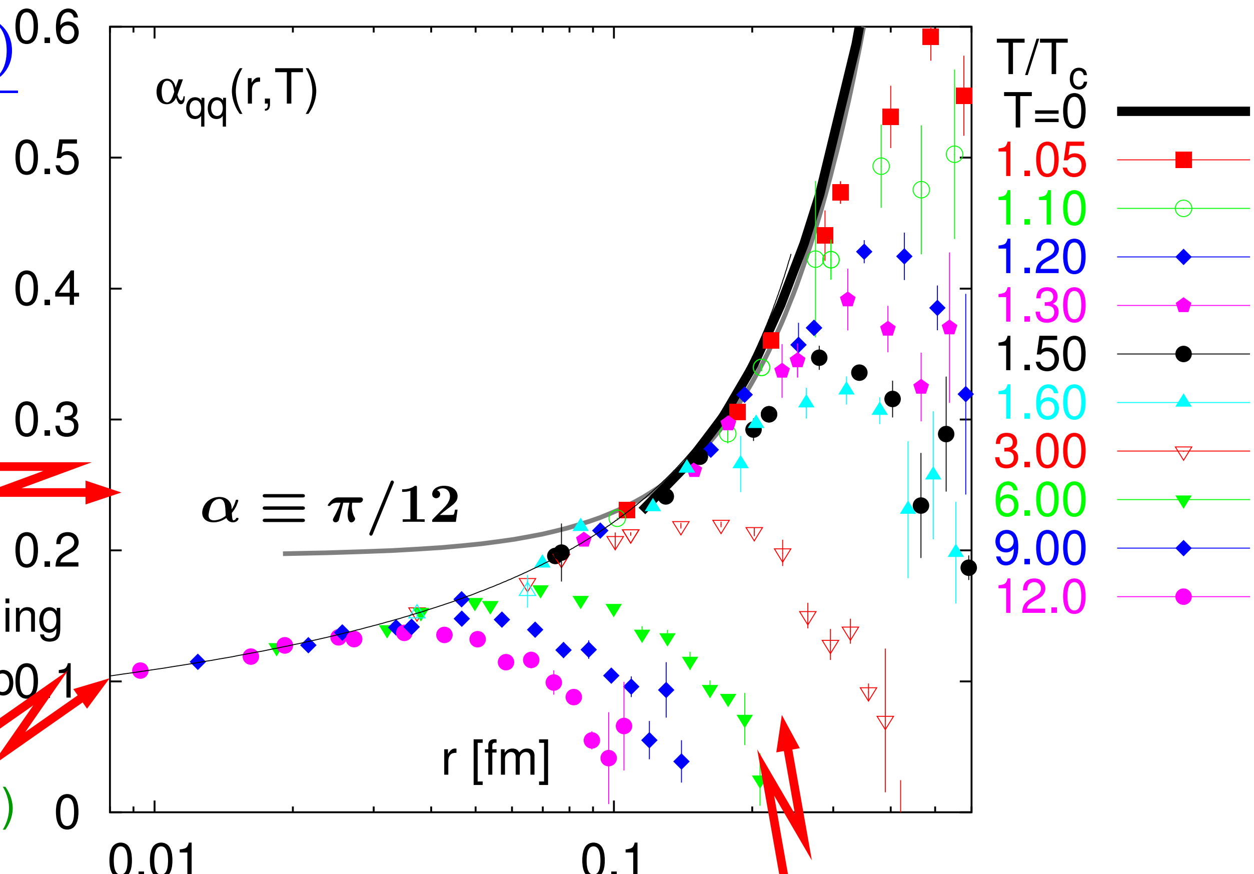


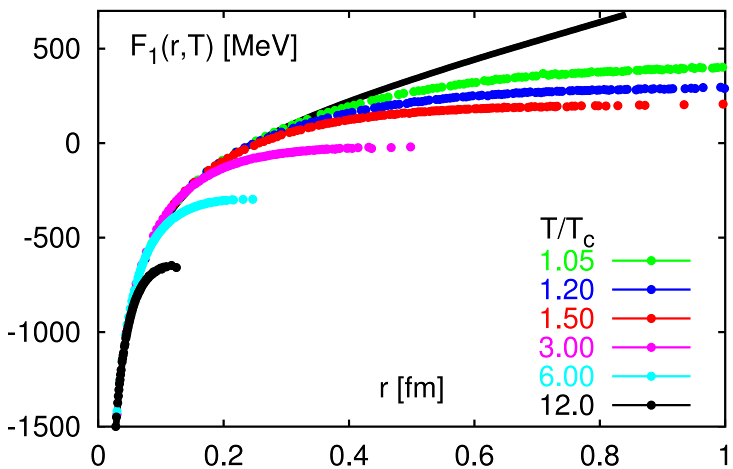
large distance: constant Coulomb term (string model)

short distance: running coupling $\alpha(r)$ from $(T=0)$, 3-loop
(S. Necco, R. Sommer, Nucl. Phys. B622 (2002) 328)

T-dependence starts in non-perturbative regime for $T \lesssim 3\ T_c$

- short distance physics ⇔ vacuum physics

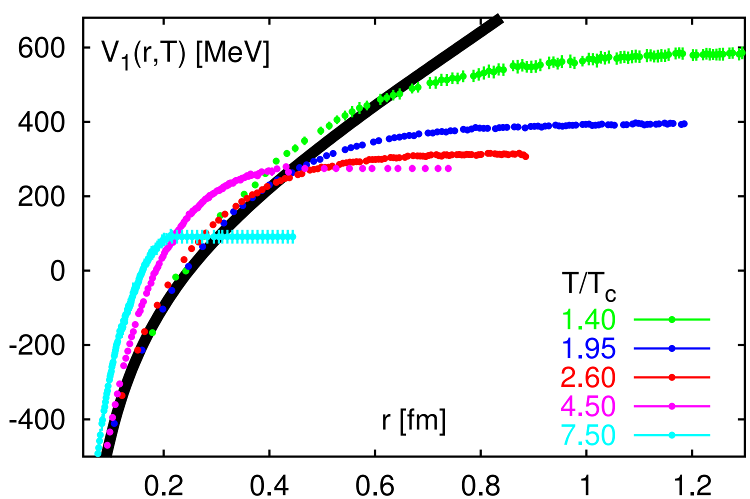
From heavy quark free energies to heavy quark potentials



i) singlet free energy

$$\exp(-F_1(r,T)/T) = \frac{1}{3}\langle \mathrm{Tr} L_{\vec{x}} L_{\vec{0}}^{\dagger} \rangle$$

(Coulomb gauge)



ii) singlet energy ⇔ "potential" energy

$$V_1(r,T) \equiv -T^2 \frac{\partial F_1(r,T)/T}{\partial T}$$

- potential is "deeper": $V(r,T) > F(r,T)$
- potential "barrier" high also well above T_c
- "potential" screened at short distances

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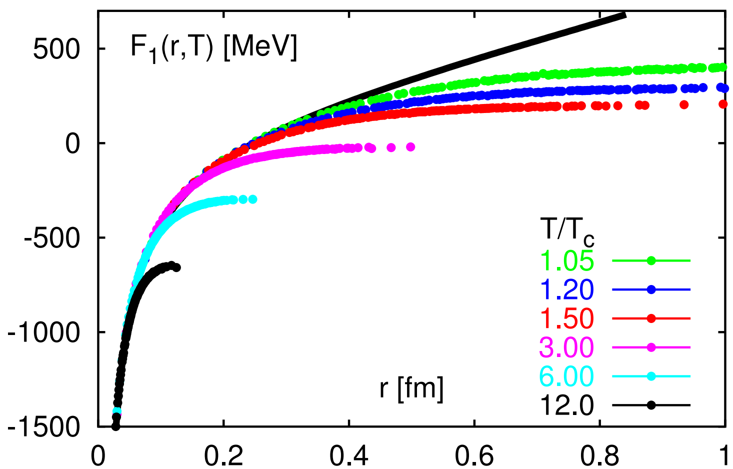
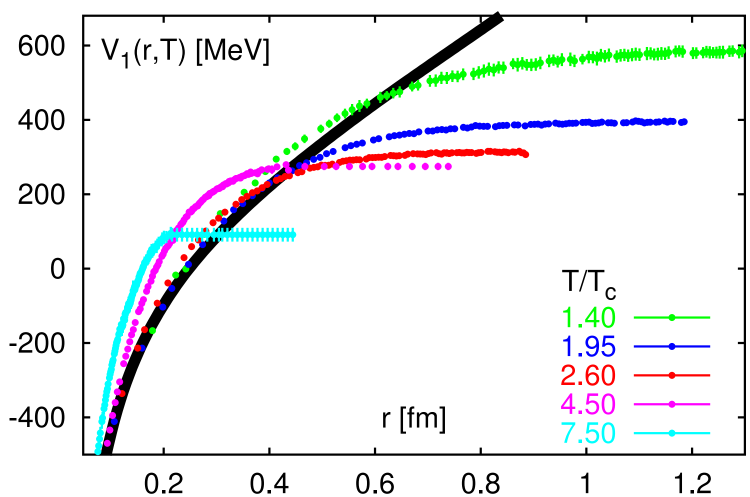


i) singlet free energy

NOTE:

$F_{\bar{q}q}(r,T)$ decreases with increasing T and fixed r $\Rightarrow$ positive entropy

$$F_1(\infty, 1.4T_c) \simeq 200 \text{ MeV}$$
$$V_1(\infty, 1.4T_c) \simeq 600 \text{ MeV}$$



ii) singlet energy $\Leftrightarrow$ "potential" energy

When do heavy quark bound states really disappear?

i) neither V_1 nor F_1 are "potentials"
ii) potential models are MODELS!

Heavy quark bound states from Schrödinger-Equation

- Schrödinger equation for heavy quarks:

$$\left[2m_a + \frac{1}{m_a}\nabla^2 + V_1(r,T)\right] \Phi_i^a = M_i^a(T)\, \Phi_i^a \quad , \quad a \;=\; \mathrm{charm,\ bottom}$$

  - T-dependent color singlet heavy quark potential mimics in-medium modification of $q\bar{q}$ interaction
  - reduction to 2-particle interaction clearly too simple, in particular close to T_c

- recent analyses:

  using F_1: S. Digal, P. Petreczky, H. Satz, Phys. Lett. B514 (2001) 57;

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state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ"
E_s^i [GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
T_d/T_c	1.1	0.74	0.1 - 0.2	2.31	1.13	1.1	0.83	0.74
T_d/T_c	$\sim$ 2.0	$\sim$ 1.1	$\sim$ 1.1	$\sim$ 4.5	$\sim$ 2.0	$\sim$ 2.0	–	–

V_1 leads to dissociation temperatures consistent with spectral function analysis

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- Schrödinger-eq. yields $T_\chi > T_{\psi'}$
- collision with thermal gluons, $\langle p \rangle \sim 3\,T$ can lead to earlier dissociation: $\mathrm{d}n_{J/\psi}/\mathrm{d}t = -n_g \langle \sigma_{dis} \rangle$

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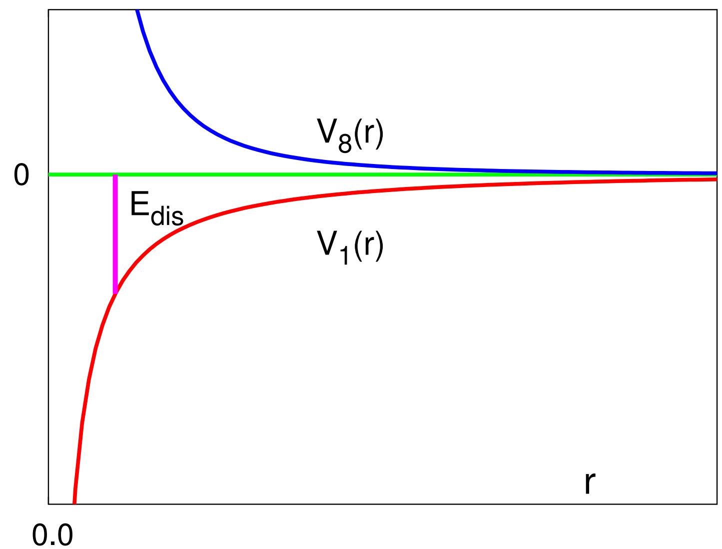

collisional dissociation
D. Kharzeev, H. Satz, PL B334 (1994) 155

$$J/\psi \; + \; g \quad \rightarrow \quad c \; + \; \bar{c}$$

$$T \; = \; 1.1\, T_c : \quad E_{dis,\chi} \simeq 50 \text{ MeV}$$

$$E_{dis,J\psi} \simeq 500 \text{ MeV}$$

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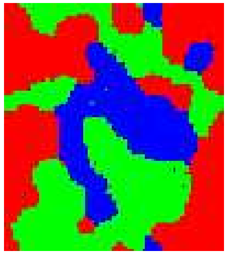
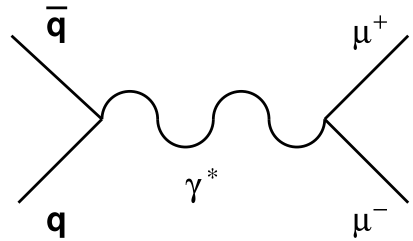


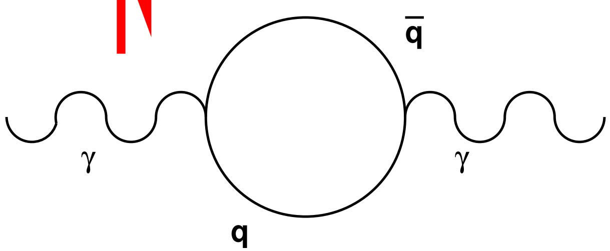
Spectral functions and Dilepton rates

Thermal dilepton rate and vector spectral function



L.D. McLerran, T. Toimela, PR D31 (85) 545.

rate $\sim |q\bar{q} \longrightarrow \gamma^*|^2 \cdot |l^+l^- \longrightarrow \gamma^*|^2$



photon self-energy

⇕

propagation of a $q\bar{q}$-pair with
the quantum numbers of a vector meson

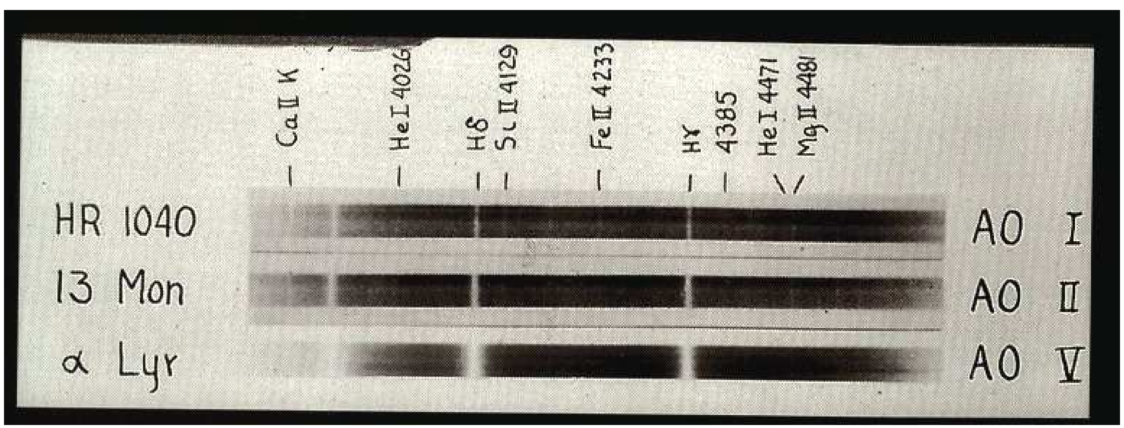
spectral representation of dilepton rate

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{1}{\omega^2 (e^{\omega/T} - 1)} \sigma_V(\omega, \vec{p}, T)$$

Spectral lines emitted by stars: pressure broadening

screening, collision/pressure broadening: $\Delta\lambda = \frac{\lambda^2 n\sigma}{\pi c}\left(\frac{2kT}{m}\right)^{1/2}$

- spectral functions incorporate excitation, dissolution and recombination of states
- stellar atmosphere modifies electric field of an emitting atom



supergiant

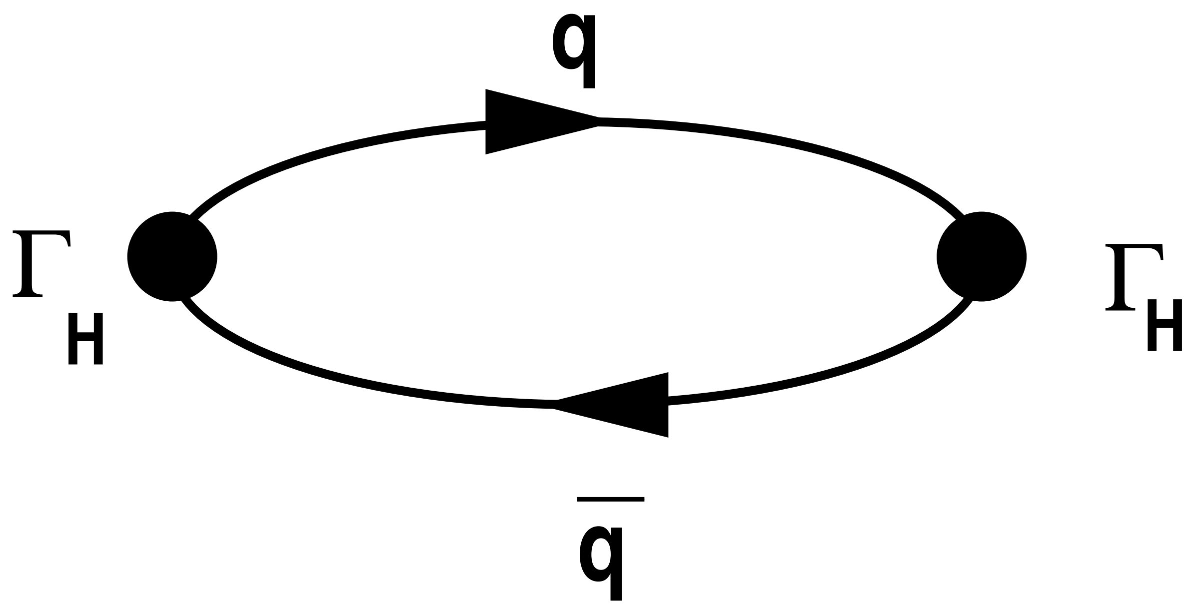
giant

dwarf

Thermal meson correlation functions and spectral functions

Thermal correlation functions: 2-point functions which describe propagation of a $\bar{q}q$-pair

spectral representation of correlator $\Rightarrow$ in-medium properties of hadrons;
thermal dilepton (photon) rates



$$G_H^\beta(\tau,\vec{r}) \;=\; \langle J_H(\tau,\vec{r}) J_H^\dagger(0,\vec{0})\rangle \;; \quad J_H(\tau,\vec{r}) = \bar{q}(\tau,\vec{r})\Gamma_H q(\tau,\vec{r})$$

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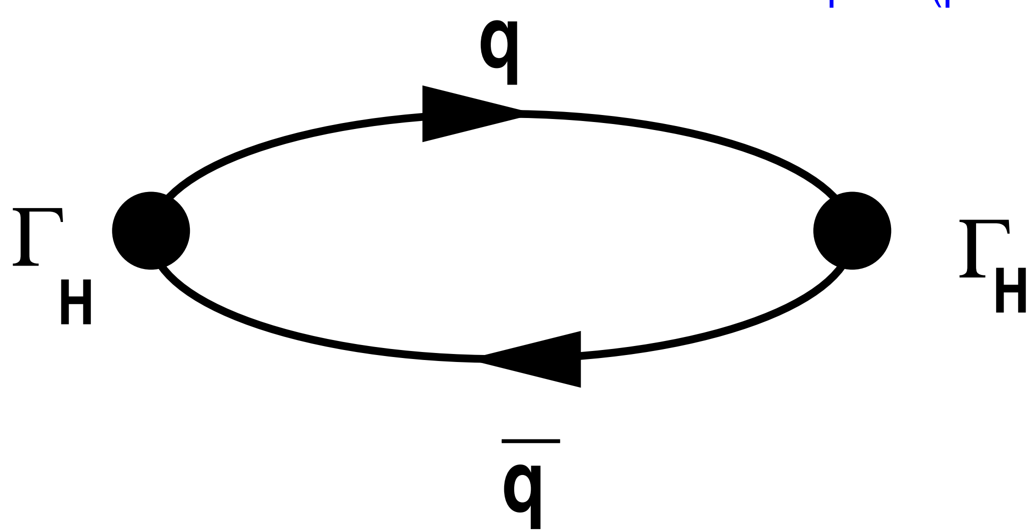

spectral representation of

Euclidean correlation functions

$$G^{\beta}_{H}(\tau,\vec{r}) = \int_0^\infty \mathrm{d}\omega \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\, \sigma_H(\omega,\vec{p},T)\, \mathrm{e}^{i\vec{p}\vec{r}}\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

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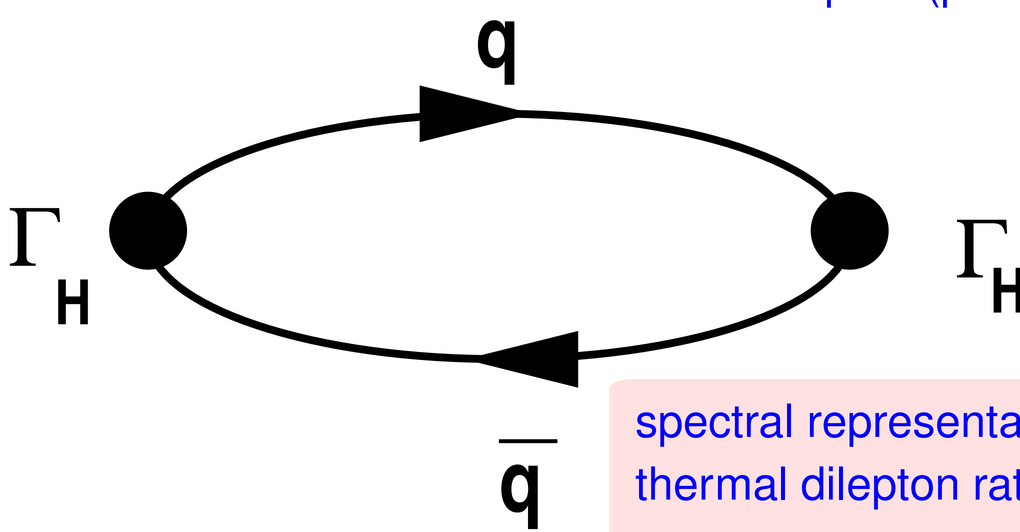


spectral representation of thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$

spectral representation of Euclidean correlation functions

$$G_H^\beta(\tau,\vec{r}) = \int_0^\infty \mathrm{d}\omega \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\, \sigma_H(\omega,\vec{p},T)\, \mathrm{e}^{i\vec{p}\vec{r}}\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

Thermal correlation functions for hadronic excitations in QCD

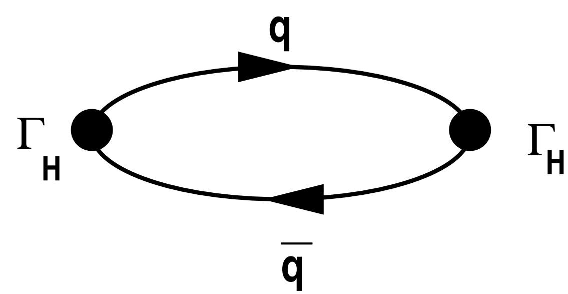
thermal modifications of the hadron spectrum is encoded in finite temperature

Euclidean correlation functions

- hadronic (mesonic) currents, composite $q\bar{q}$-operators

$$J_H = \bar{\psi}(\tau,\vec{r})\ \Gamma_H\ \psi(\tau,\vec{r})$$

- $G_H^\beta(\tau,\vec{r}) \ \equiv\ \langle J_H(\tau,\vec{r}) J_H^\dagger(0,\vec{0})\rangle_\beta$



- quantum numbers (H) fixed through Γ_H:

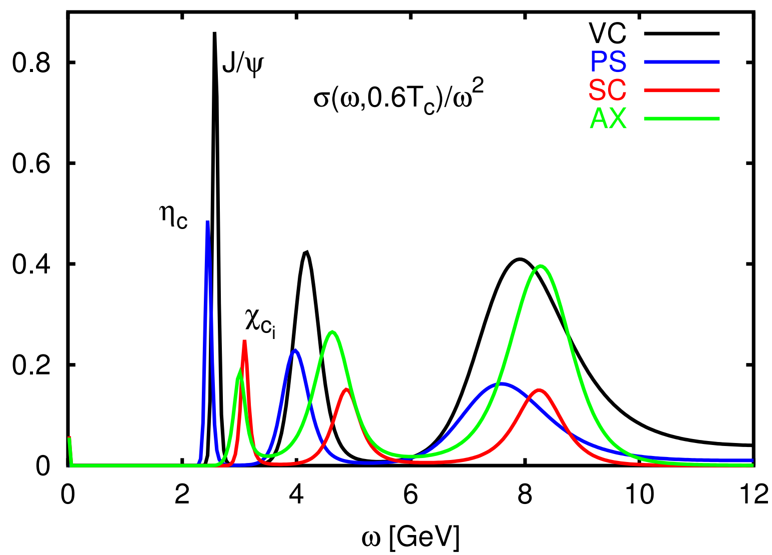
state		J^{PC}	Γ_H	(u,d)-states	$c\bar{c}$-states	$b\bar{b}$-states
scalar	$^3P_0$	0^{++}	1	σ	χ_{c0}	χ_{b0}
pseudo-scalar	$^1S_0$	0^{-+}	γ_5	π	η_c	η_b
vector	$^3S_1$	1^{--}	γ_μ	ρ	J/ψ	Υ
axial-vector	$^3P_1$	1^{++}	$\gamma_\mu\gamma_5$	δ	χ_{c1}	χ_{b1}

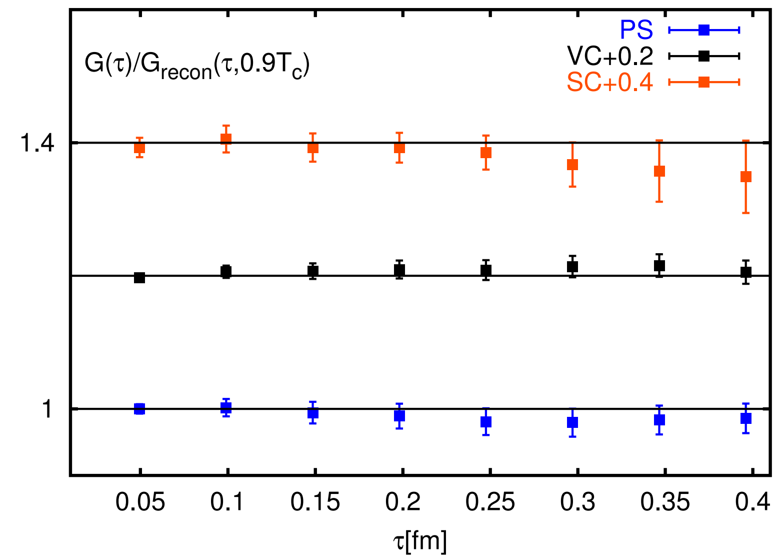
- so far all studies have been performed in quenched QCD (reasonable) with Wilson fermions (poor resolution at high energies)

Heavy quark spectral functions and correlation functions

- left: charmonium spectral functions below T_c, *i.e.* at $T \simeq 0.6\ T_c$, lattice size $48^3 \times 24$
- right: correlation function at $T = 0.9T_c$ over reconstructed correlation function at $T \simeq 0.9\ T_c$ using the spectral function generated at $T \simeq 0.6\ T_c$, *i.e.*

$$G_{recon}(\tau, 0.9T_c) = \int \mathrm{d}\omega\ \sigma(\omega, 0.6T_c) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$



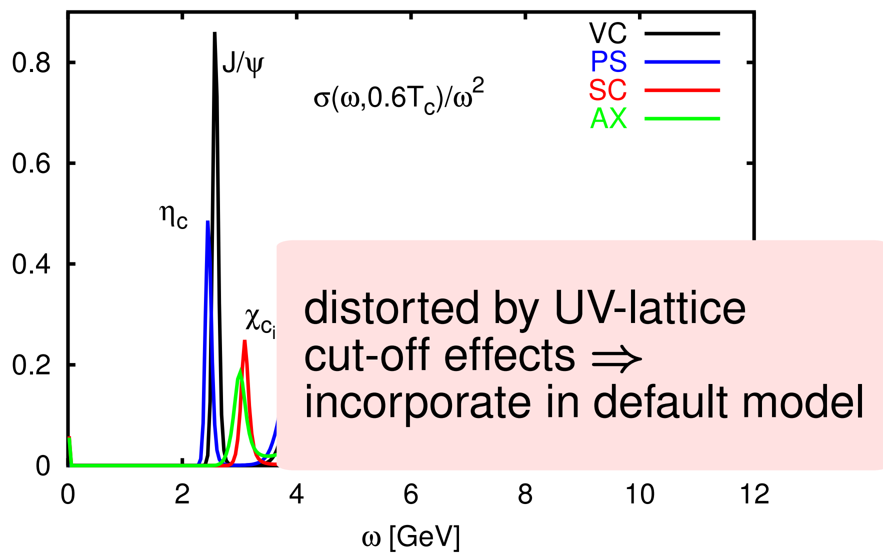


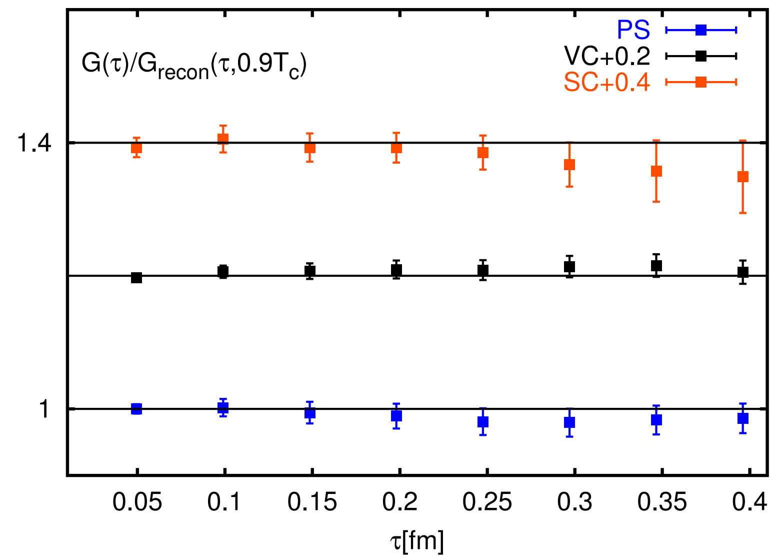
⇒ no significant temperature dependence below T_c

Heavy quark spectral functions and correlation functions

- left: charmonium spectral functions below T_c, *i.e.* at $T \simeq 0.6\ T_c$, lattice size $48^3 \times 24$
- right: correlation function at $T = 0.9T_c$ over reconstructed correlation function at $T \simeq 0.9\ T_c$ using the spectral function generated at $T \simeq 0.6\ T_c$, *i.e.*

$$G_{recon}(\tau, 0.9T_c) = \int \mathrm{d}\omega\ \sigma(\omega, 0.6T_c) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

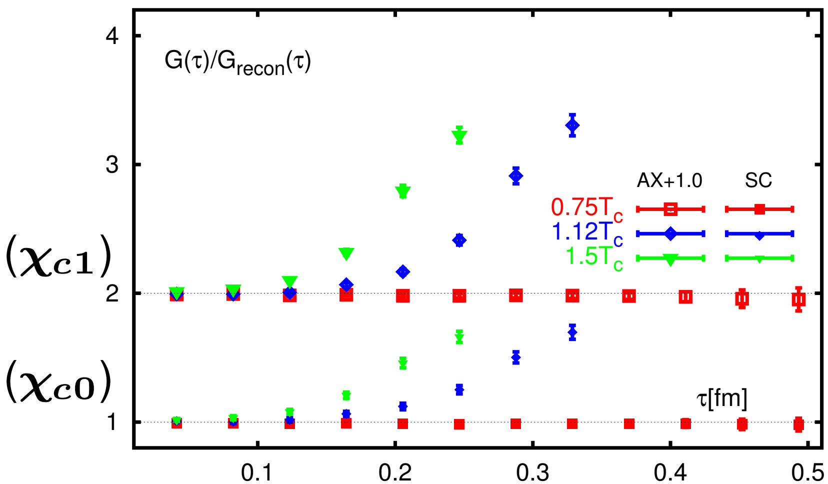




⇒ no significant temperature dependence below T_c

Heavy quark spectral functions and correlation functions

data for $G_H(\tau, T)$ over reconstructed correlation functions at T from data below T_c

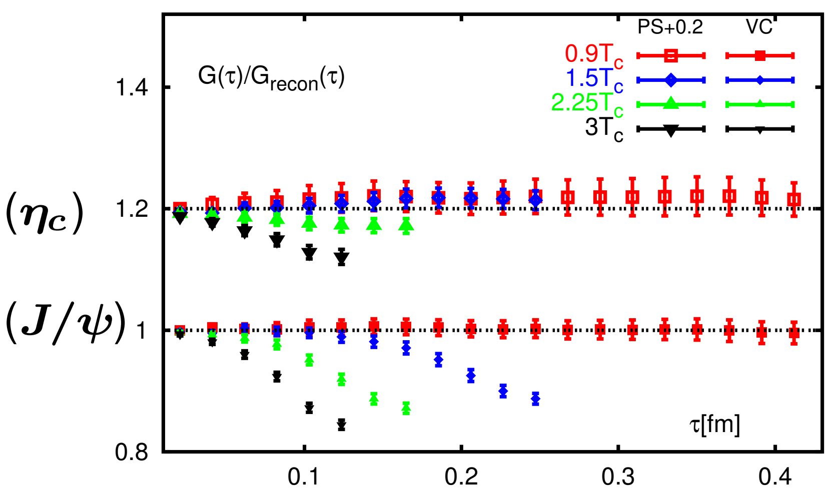


scalar and axial-vector correlation functions:

strong temperature dependence just above T_c for χ_c states

(normalized at $T < T_c$)

($48^3 \times N_\tau,\ N_\tau = 12,\ 16,\ 24,\ a = 0.04$ fm)



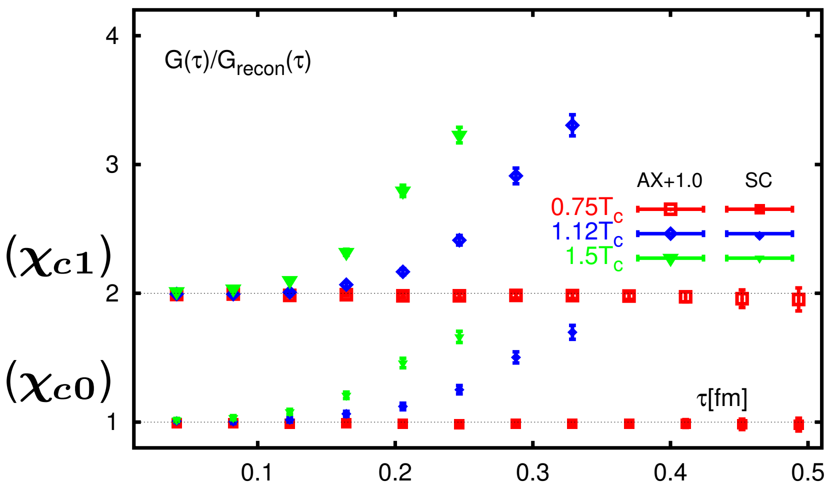
vector and pseudoscalar correlation functions:

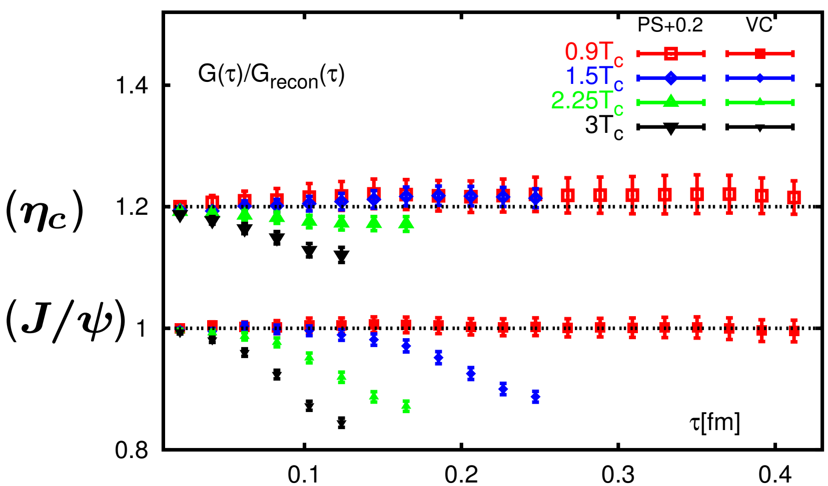
no temperature dependence for η_c up to $1.5\ T_c$; only mild but systematic temperature dependence of J/ψ

(normalized at $T < T_c$)

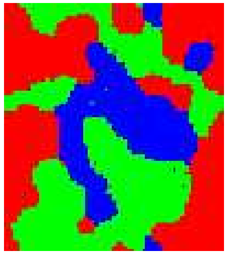
($N_\sigma = 40,\ 48,\ 64,$
$N_\tau = 12,\ 16,\ 24,\ 40,\ a = 0.02$ fm)

Heavy quark spectral functions and correlation functions

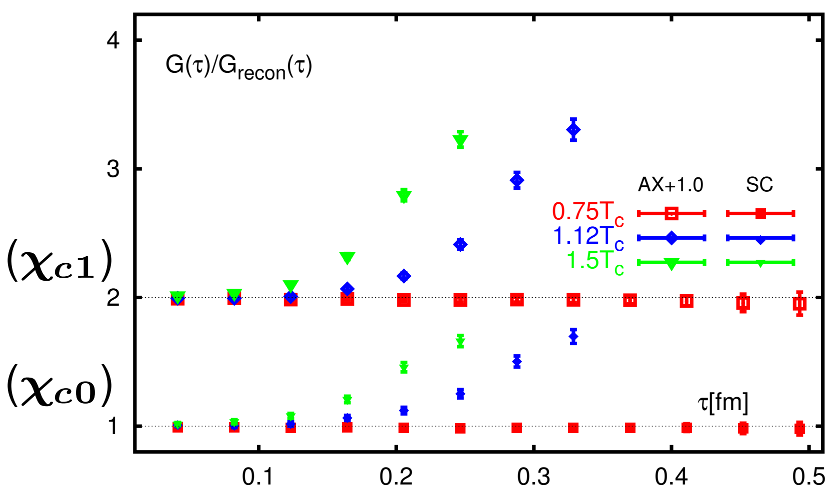




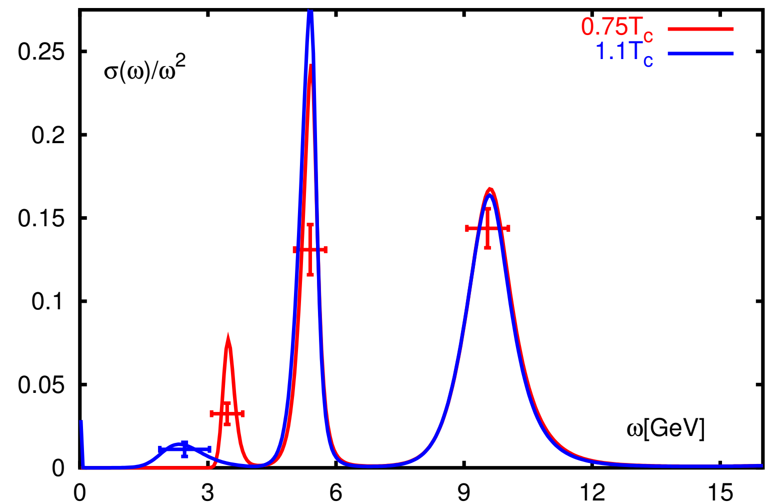
pattern seen in correlation functions also visible in spectral functions

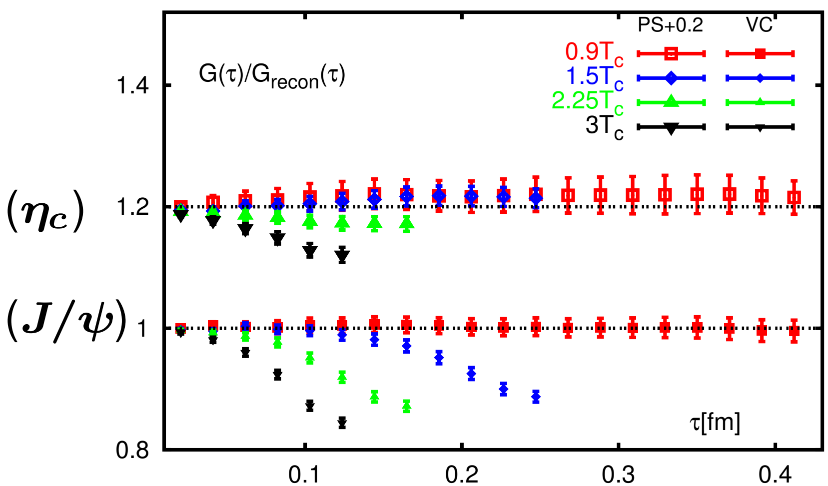


Heavy quark spectral functions and correlation functions

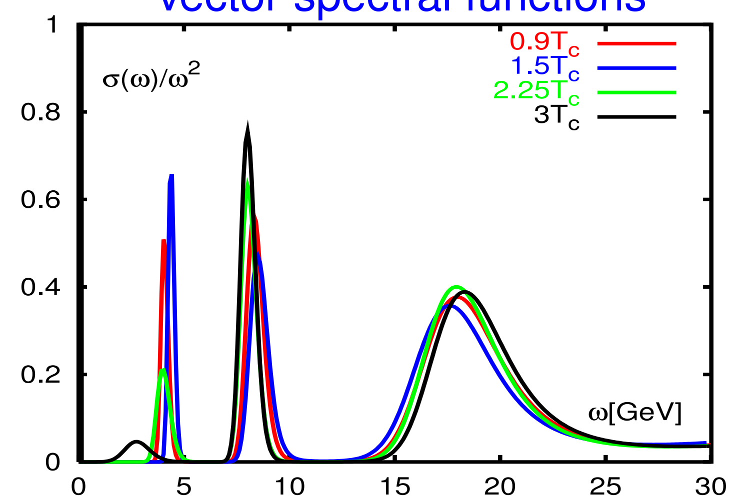


scalar spectral functions

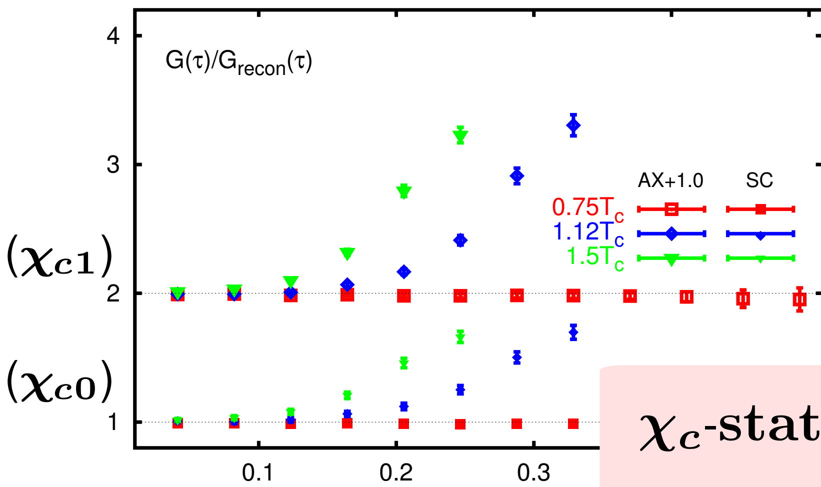




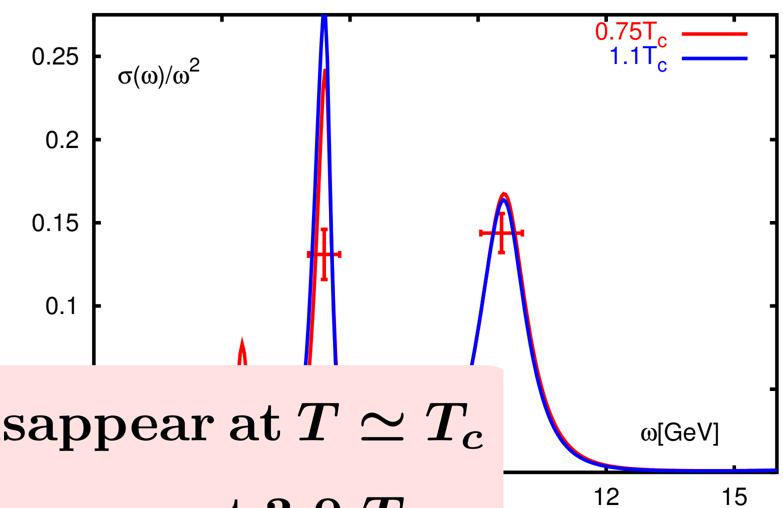
vector spectral functions



Heavy quark spectral functions and correlation functions

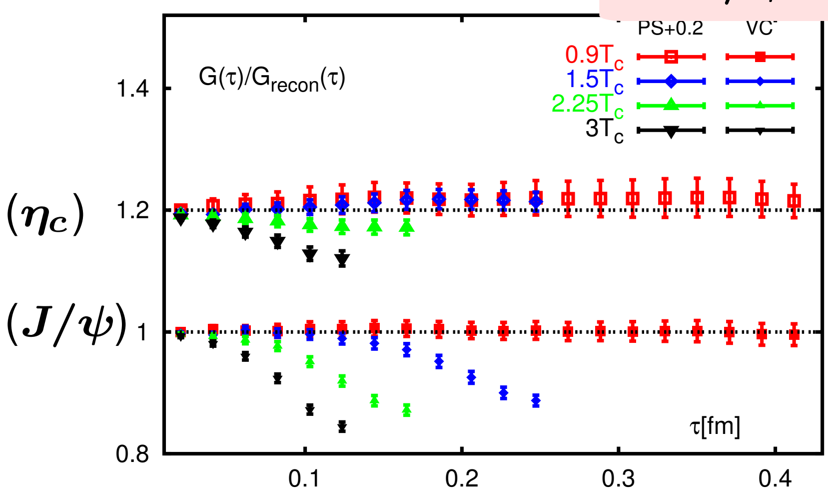


scalar spectral functions

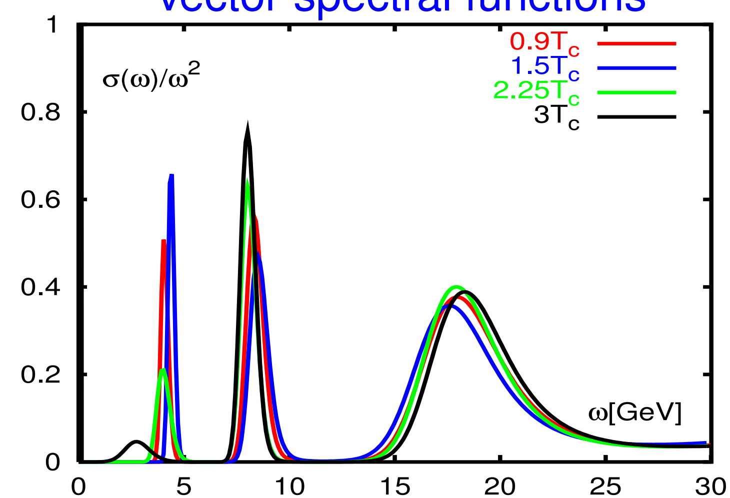


χ_c-states disappear at $T \simeq T_c$

J/ψ and η_c gone at $3.0\ T_c$



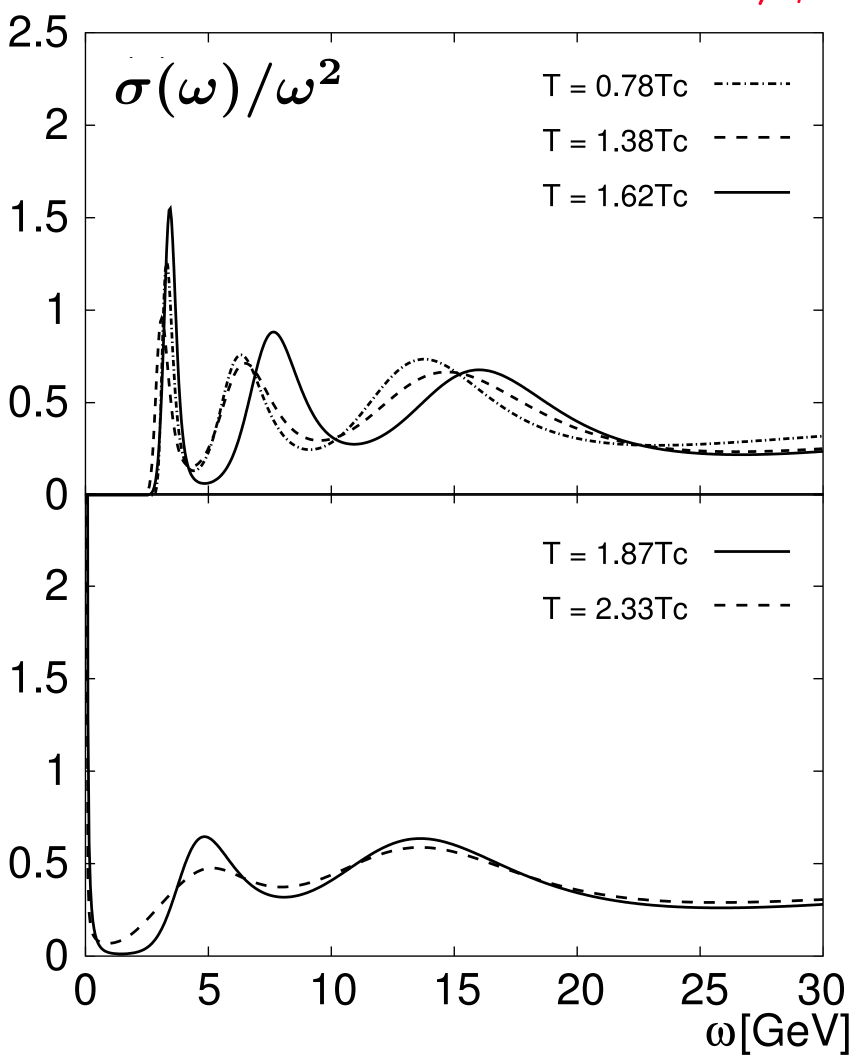
vector spectral functions

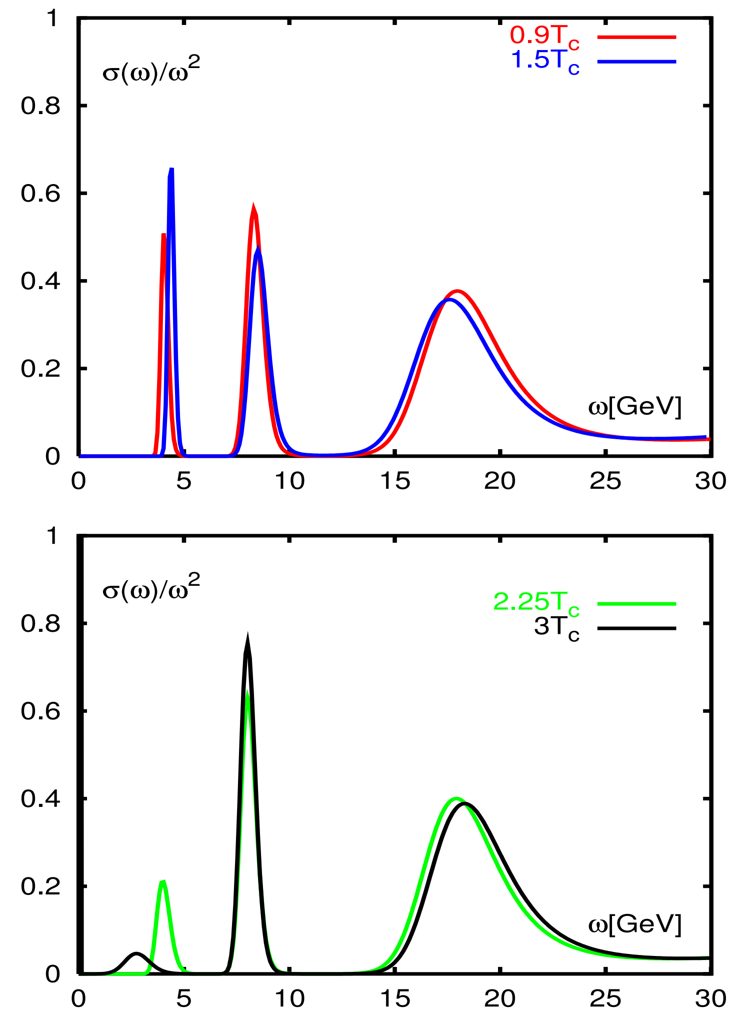


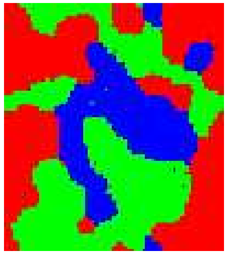
Heavy quark spectral functions
comparison of different approaches

M. Asakawa, T. Hatsuda, hep-lat/0308034 S. Datta et al., hep-lat/0312037

J/ψ spectral function







Heavy quark spectral functions
comparison of different approaches

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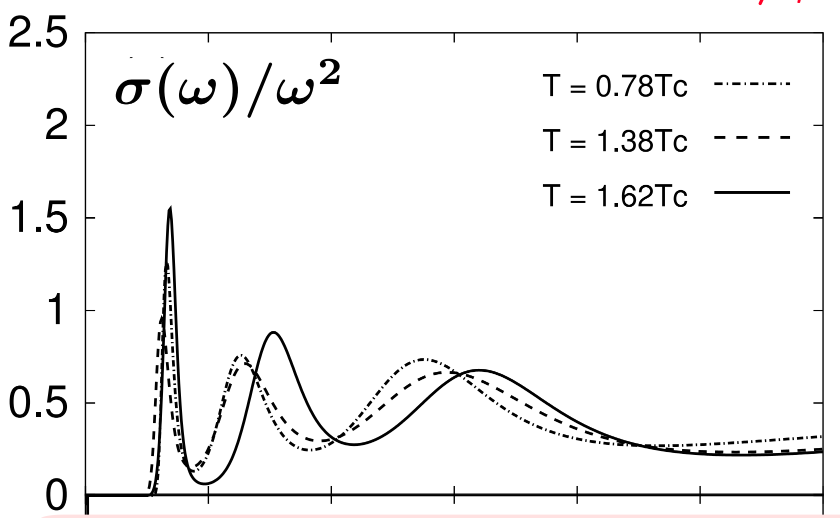
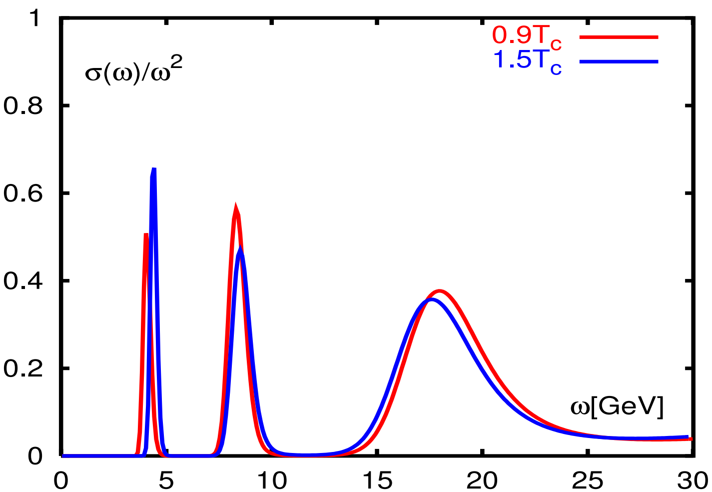
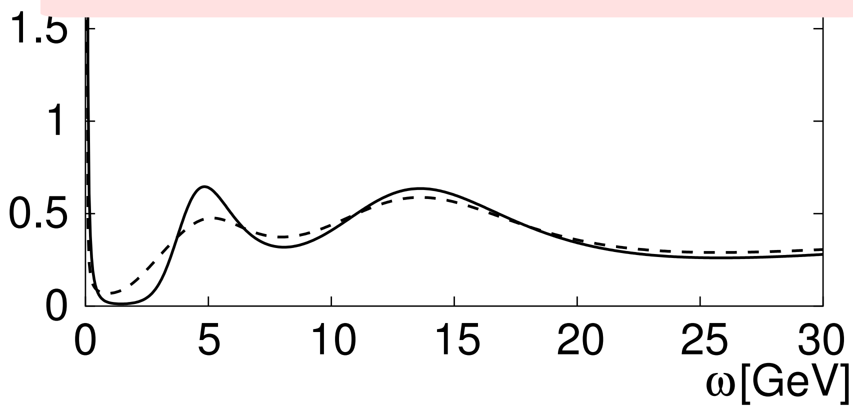
J/ψ spectral function

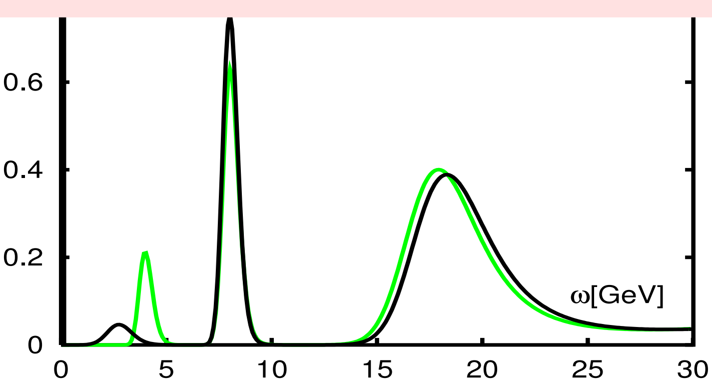




J/ψ dissociates for $1.6T_c \lesssim T \lesssim 1.9T_c$
rather abrupt disappearance of J/ψ

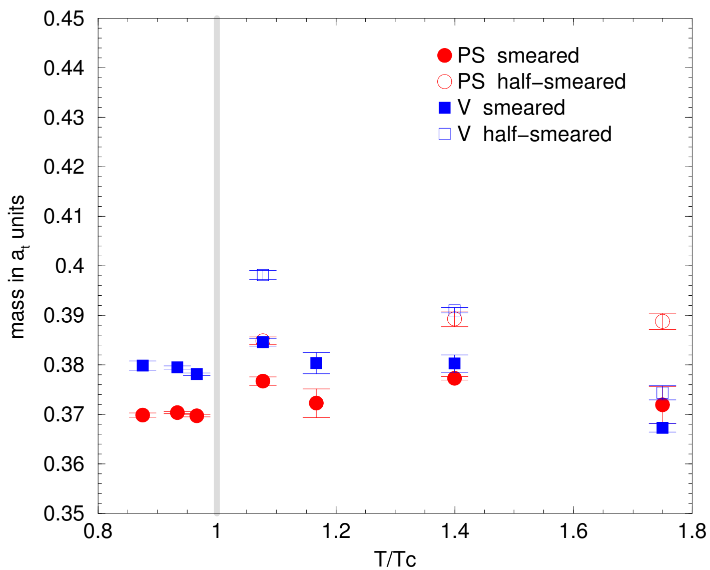
J/ψ gradually disappears for $T \gtrsim 1.5T_c$
J/ψ strength reduced by 25% at $T = 2.25T_c$

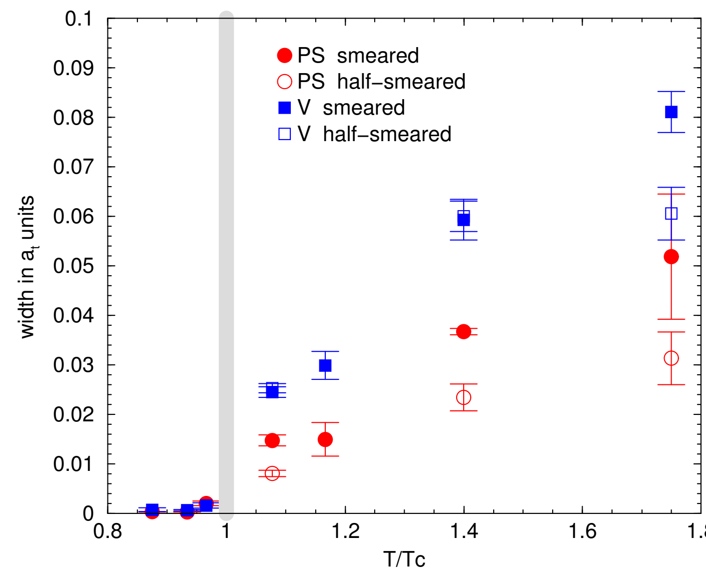




Heavy quark spectral functions
pressure broadening

- thermal broadening of charmonium spectral functions?
- no ”first principle” evidence, BUT some evidence using resonance ansatz that incorporates a thermal width





T. Umeda, Proceedings of the RIKEN-BNL workshop on Lattice QCD at finite temperature and density, BNL-72083-2004

Heavy quark spectral functions: bottomonium

- first results on bottomonium spectral functions using MEM on anisotropic lattices

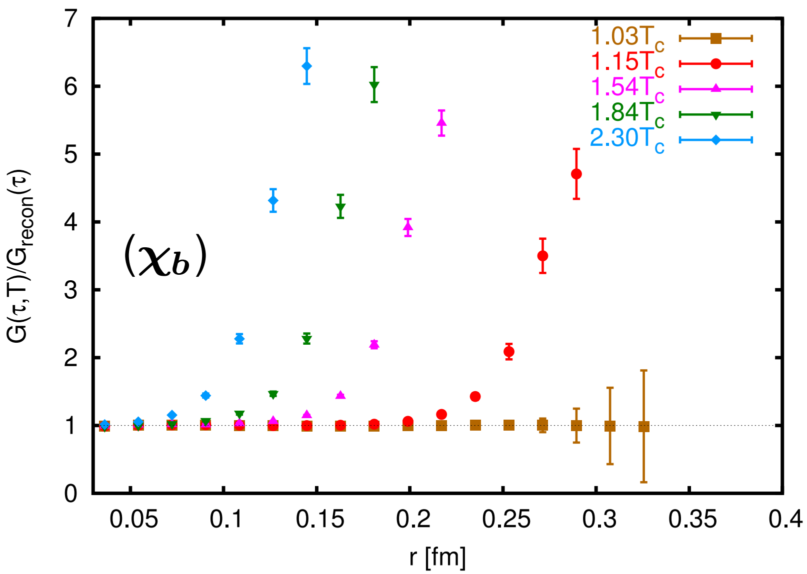
  A. Jakovac, P. Petreczky, K. Petrov and A.Velytsky, in preparation

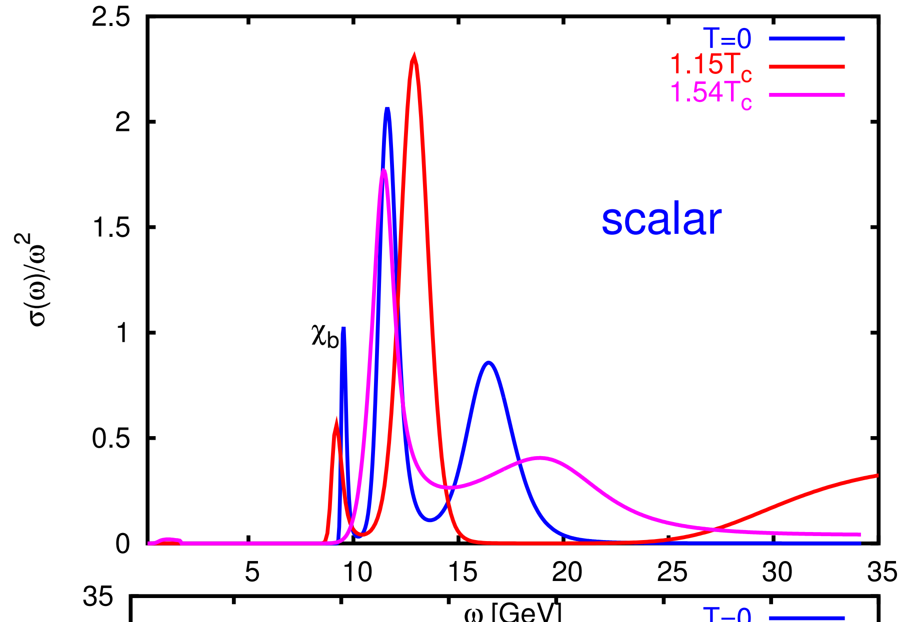
  K. Petrov, Hard Probes 2004, hep-lat/0503002

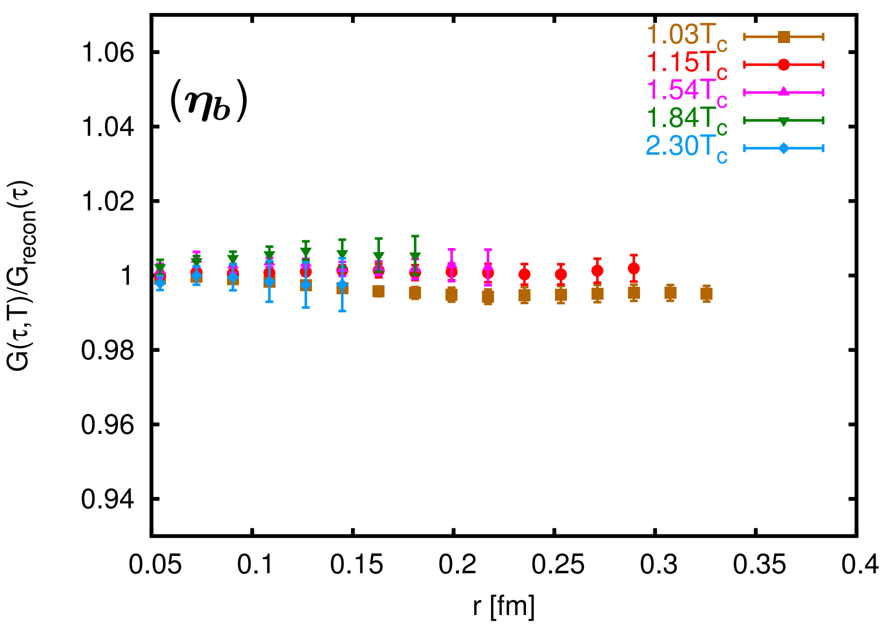
- in general: heavy states ⇒ larger discretization errors
  ⇒ finer lattices are needed

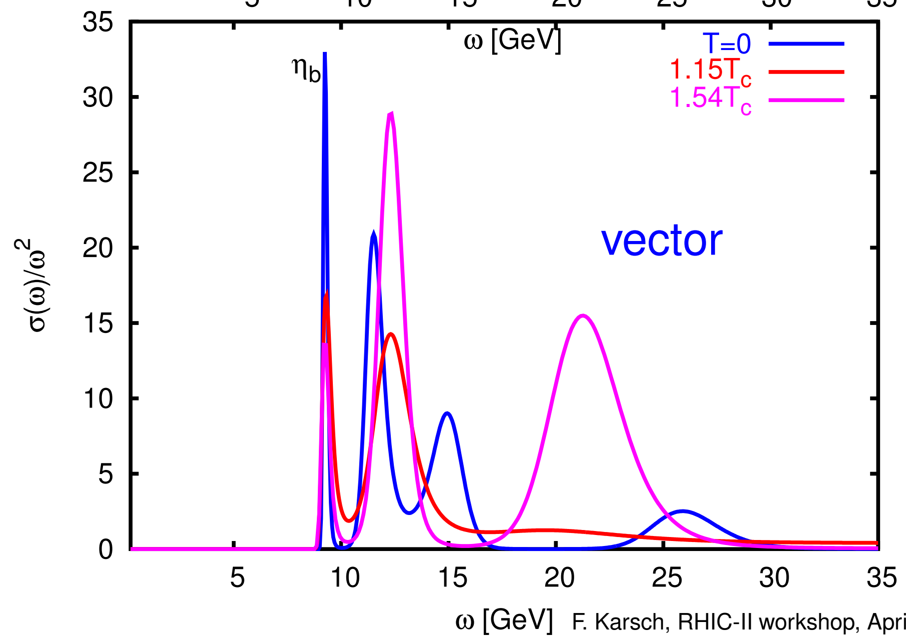
- observed pattern follows pattern observed in the charmonium systems:
  radial excitations (χ_b) are suppressed close to T_c, Υ and η_b survive beyond $2T_c$

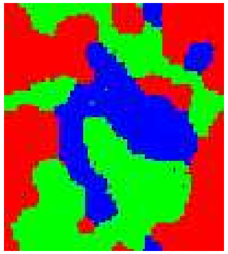
Heavy quark spectral functions: bottomonium



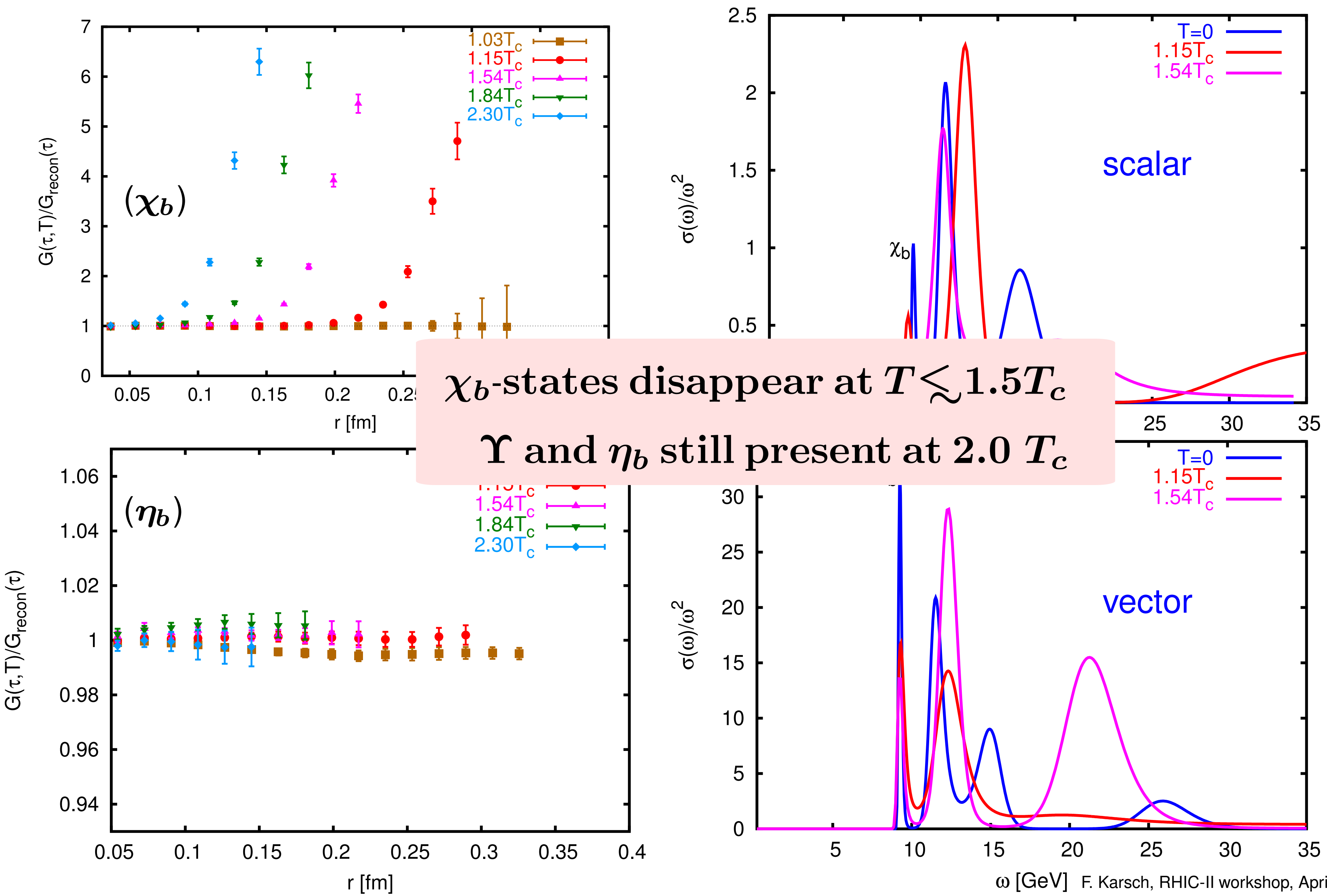








Heavy quark spectral functions: bottomonium



χ_b-states disappear at $T \lesssim 1.5 T_c$

Υ and η_b still present at 2.0 T_c

Concluding remarks

- Matsui-Satz: dissolved $c\bar{c}$ never recombine again;

  potential model approach suggests sequential suppression pattern

  - details depend on ”potential” used in Schrödinger equation
  - generic features consistent with spectral function studies
    - J/ψ survives the deconfinement transition and melts only at

$$T_{J/\psi}/T_c \sim (1.5 - 2.5)$$

    - ψ' and χ_c dissolve at (or close to) T_c

$$T_{\psi'} \; < \; T_\chi \text{ and } T_\chi \gtrsim T_c \text{ ???}$$

If so: small variations in dissociation temperature close to T_c will have significant effect on suppression pattern

- copious production of $\bar{c}c$-pairs at RHIC may allow for recombination and/or statistical hadronization
  - this may wash out sequential suppression pattern