GPDs and SSA

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Outline

- GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs
  - $H(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow q(x,\mathbf{b}_\perp)$
  - $\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x,\mathbf{b}_\perp)$
  - $E(x,0,-\mathbf{\Delta}_\perp^2)$
    $\hookrightarrow \perp$ deformation of unpol. PDFs in $\perp$ pol. target
    - Sivers effect
  - $2\tilde{H}_T + E_T \longrightarrow \perp$ deformation of $\perp$ pol. PDFs in unpol. target
    - correlation between quark angular momentum and quark transversity
    - Boer-Mulders function $h_1^\perp(x,\mathbf{k}_\perp)$
- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$

$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

Generalized Parton Distributions (GPDs)

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

- GPDs are **form factor** for only those quarks in the nucleon carrying a certain **fixed momentum fraction** x

$\hookrightarrow$ t dependence of GPDs for fixed x, provides information on the **position space distribution** of quarks carrying a certain momentum fraction x

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

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$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

M.B., PRD **62**, 71503 (2000)

- define state that is localized in $\perp$ position:
  [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\right\rangle \equiv \mathcal{N}\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp,\lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+}\int dx^- d^2\mathbf{x}_\perp\, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x,\mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi}\left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

GPDs $\longleftrightarrow$ $q(x, \mathbf{b}_\perp)$

- nucleon-helicity nonflip GPDs can be related to distribution of partons in $\perp$ plane

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2)$$

$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2)$$

- **no rel. corrections** to this result! (Galilean subgroup of $\perp$ boosts)
- $q(x, \mathbf{b}_\perp)$ has **probabilistic interpretation**, e.g.

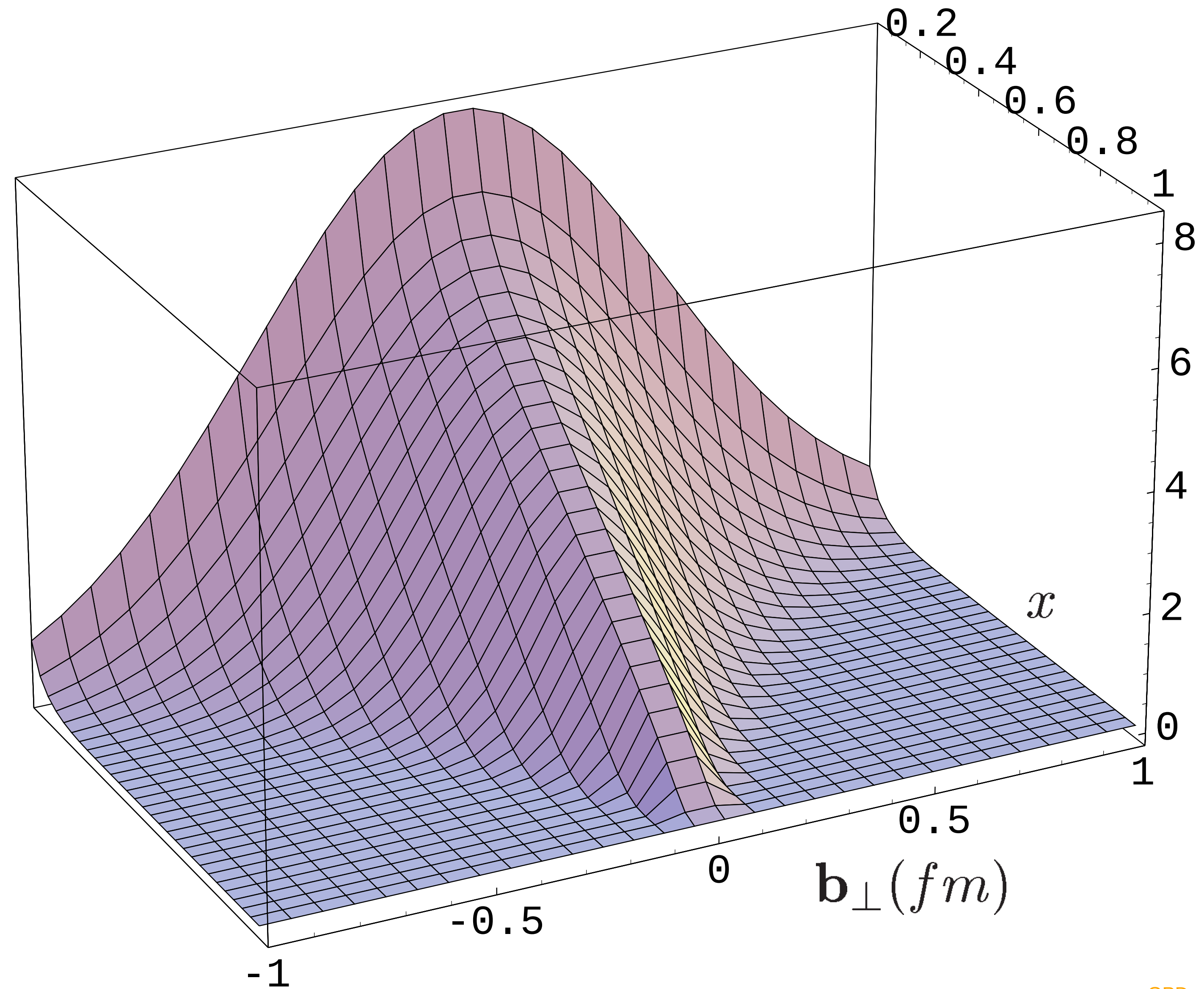
$$q(x, \mathbf{b}_\perp) \geq |\Delta q(x, \mathbf{b}_\perp)| \geq 0 \quad \text{for} \quad x > 0$$

$$q(x, \mathbf{b}_\perp) \leq -|\Delta q(x, \mathbf{b}_\perp)| \leq 0 \quad \text{for} \quad x < 0$$

GPDs $\longleftrightarrow$ $q(x, \mathbf{b}_\perp)$

- $\mathbf{b}_\perp$ distribution measured w.r.t. $\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{i,\perp}$
  $\hookrightarrow$ width of the $\mathbf{b}_\perp$ distribution should go to zero as $x \rightarrow 1$, since the active quark becomes the $\perp$ center of momentum in that limit!
  $\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp^2$-indep. as $x \rightarrow 1$. Confirmed by recent lattice studies (QCDSF, LHPC)
- Anticipated shape of $q(x, \mathbf{b}_\perp)$:
  large x: quarks from localized valence 'core',
  small x: contributions from larger ' meson cloud'
  $\hookrightarrow$ expect a gradual increase of the t-dependence ($\perp$ size) of $H(x, 0, t)$ as x decreases

$q(x, \mathbf{b}_\perp)$ in a simple model



Transversely Distorted Distributions and $E(x,0,-\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\uparrow\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$
$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\uparrow\rangle + |p^+,\mathbf{R}_\perp = \mathbf{0}_\perp,\downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **78**, 610 (2003)]

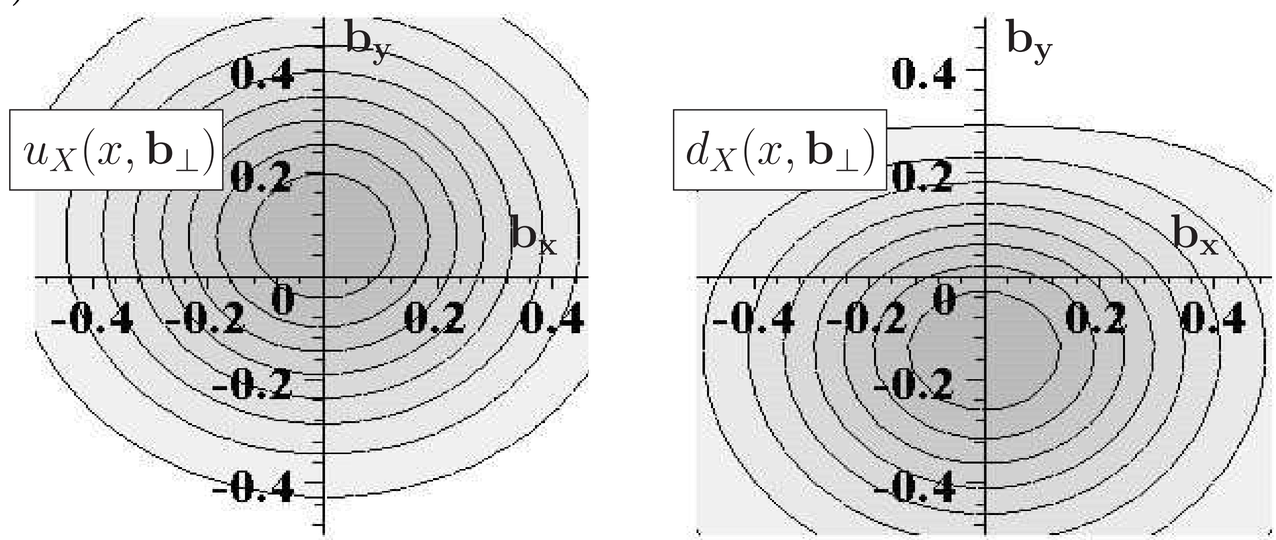
Transversely Distorted Distributions and $E(x, 0, -\Delta_\perp^2)$

- $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized nucleon is distorted compared to longitudinally polarized nucleons !
- mean $\perp$ displacement of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

Here $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2)$ and $\kappa_p = \frac{2}{3}\kappa_p^u - \frac{1}{3}\kappa_p^d = 1.793$

$\hookrightarrow$ $d_y^q = \mathcal{O}(0.2 fm)$



GPD $\longleftrightarrow$ SSA (Sivers)

- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- without FSI, $\langle \mathbf{k}_\perp \rangle = 0$, i.e. $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) = 0$
- with FSI, $\langle \mathbf{k}_\perp \rangle \neq 0$ (Brodsky, Hwang, Schmidt)
- FSI formally included by appropriate choice of Wilson line gauge links in gauge invariant def. of $q(x,\mathbf{k}_\perp)$

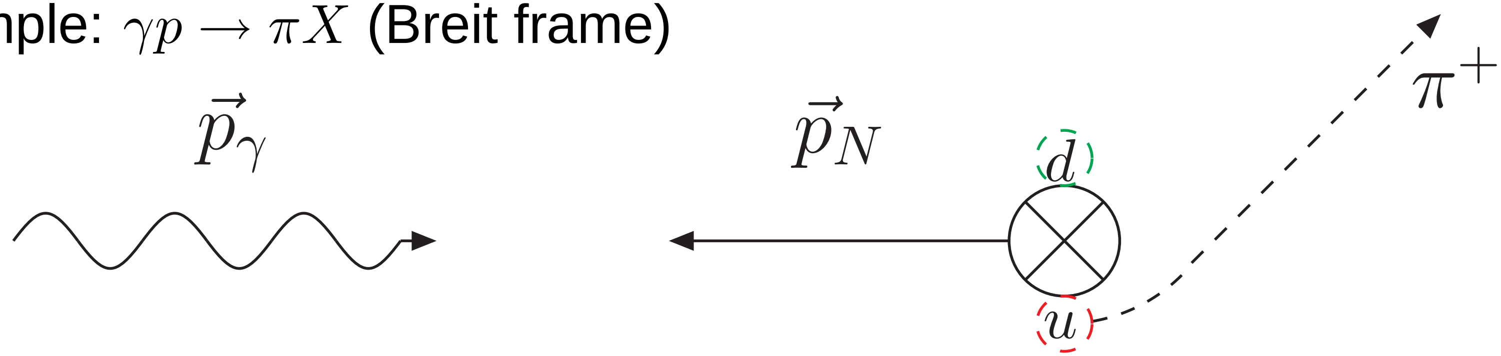
$\hookrightarrow$ Qiu, Sterman; Collins; Ji; Boer et al.;..

$$\langle \mathbf{k}_\perp \rangle \sim \left\langle P,S \left| \bar{q}(0)\gamma^+ \int_0^\infty d\eta^- G^{+\perp}(\eta) q(0) \right| P,S \right\rangle$$

  - $\int_0^\infty d\eta^- G^{+\perp}(\eta)$ is the $\perp$ impulse that the active quark acquires as it moves through color field of “spectators”
- What should we expect for Sivers effect in QCD ?

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign determined by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q$

- $f_{1T}^{\perp q} \sim -\kappa_q$ consistent with HERMES results

GPD $\longleftrightarrow$ SSA (Sivers); formal argument

- treat FSI to lowest order in g

$\hookrightarrow$

$$\langle k_q^i \rangle = -\frac{g}{4p^+}\int \frac{d^2\mathbf{b}_\perp}{2\pi}\frac{b^i}{|\mathbf{b}_\perp|^2}\left\langle p,s\left|\bar{q}(0)\gamma^+\frac{\lambda_a}{2}q(0)\rho_a(\mathbf{b}_\perp)\right|p,s\right\rangle$$

with $\rho_a(\mathbf{b}_\perp)=\int dr^-\rho_a(r^-,\mathbf{b}_\perp)$ summed over all quarks and gluons

$\hookrightarrow$ SSA related to dipole moment of density-density correlations

- GPDs (N polarized in $+\hat{x}$ direction): $u \longrightarrow +\hat{y}$ and $d \longrightarrow -\hat{y}$

$\hookrightarrow$ expect density density correlation to show same asymmetry $\langle b^y\bar{u}(0)\gamma^+\frac{\lambda_a}{2}u(0)\rho_a(\mathbf{b}_\perp)\rangle > 0$

$\hookrightarrow$ sign of SSA opposite to sign of distortion in position space

Chirally Odd GPDs

$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \sigma^{+j}\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = \quad H_T \bar{u}\sigma^{+j}\gamma_5 u + \tilde{H}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2} u$$
$$+ E_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta} P_\alpha \gamma_\beta}{M}$$

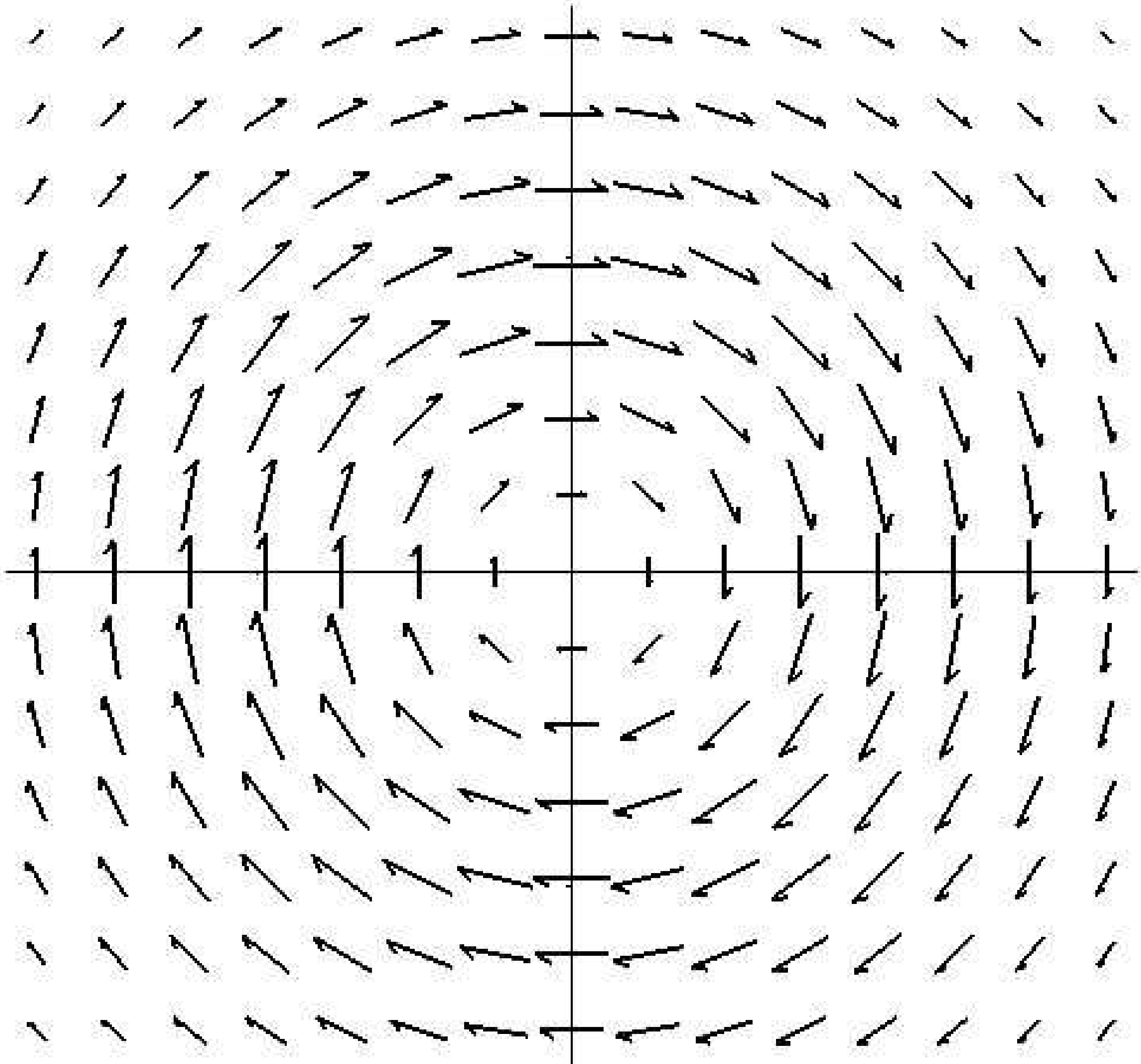
- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of $2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity for unpolarized target in $\perp$ plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M}\frac{\partial}{\partial b_j} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \left[2\tilde{H}_T^q(x, 0, -\mathbf{\Delta}_\perp^2) + E_T^q(x, 0, -\mathbf{\Delta}_\perp^2)\right]$$

- origin: correlation between quark spin (i.e. transversity) and angular momentum
- angular momentum J_q^i carried by quarks with transverse spin s^j in an unpolarized target

$$\langle J_q^i(s^j)\rangle = \frac{\delta_{ij}}{4} \int dx \left[2\tilde{H}_T(x, 0, 0) + E_T(x, 0, 0)\right] x$$

Transversity Distribution in Unpolarized Target



Transversity Decomposition of J_q (a cool new "sum rule

- project on $\perp$ spin (transversity) in $\hat{y}$ direction $P_{\pm\hat{y}} \equiv \frac{1}{2}\left(1 \mp \gamma_5\gamma^y\right)$
- Neither T^{0z} nor T^{0x} mix eigenstates of $P_{\pm\hat{y}}$

$\hookrightarrow$ decompose $J_q^y \equiv \int d^3x \left(T_q^{0x}z - T_q^{0z}x\right)$ w.r.t. transverse polarization (transversity) of quarks $J_q^y = J_{q,+\hat{y}}^y + J_{q,-\hat{y}}^y$

- use (M.Diehl and P.Hägler,hep-ph/0504175)

$$\langle p'|\,\bar{q}\sigma^{\lambda\mu}\gamma_5 i\overset{\leftrightarrow}{D}{}^{\nu}\,q\,|p\rangle = \bar{u}\sigma^{\lambda\mu}\gamma_5 u\,\bar{p}^{\nu} A_{T20}(t) + \frac{\varepsilon^{\lambda\mu\alpha\beta}\Delta_\alpha\bar{p}_\beta\bar{p}^\nu}{M^2}\bar{u}u\,\tilde{A}_{T20}(t)$$
$$+\frac{\varepsilon^{\lambda\mu\alpha\beta}\Delta_\alpha\bar{p}^\nu}{2M}\bar{u}\gamma_\beta u\,B_{T20}(t) + \frac{\varepsilon^{\lambda\mu\alpha\beta}\bar{p}_\alpha\Delta^\nu}{M}\bar{u}\gamma_\beta u\,\tilde{B}_{T21}(t)$$

- with $A_{T20}(t) = \int_{-1}^{1} dx x H_T(x,\xi,t)$, $\quad \tilde{A}_{T20}(t) = \int_{-1}^{1} dx x \tilde{H}_T(x,\xi,t)$,

$\hookrightarrow$ $B_{T20}(t) = \int_{-1}^{1} dx x E_T(x,\xi,t)$, and $-2\xi\tilde{B}_{T21}(t) = \int_{-1}^{1} dx x \tilde{E}_T(x,\xi,t)$

$$\delta^y J_q^y \equiv J_{q,+\hat{y}}^y - J_{q,-\hat{y}}^y = \frac{1}{2}\int dx\left[2\tilde{H}_T(x,0,0) + E_T(x,0,0)\right]x$$

Transversity Decomposition of J_q (“intuitive” derivation

- $J_q^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x \left[T_q^{0j}x^k - T_q^{0k}x^j\right]$

$\hookrightarrow$ $\left\langle J_q^y \right\rangle = \int d^3x \left\langle T_q^{++} \cdot x \right\rangle$

- $T_q^{++} = \bar{q}\gamma^+ \overset{\leftrightarrow}{D}{}^+ q = \sum_{\pm s_y} T_{q,s_y}^{++}$ diagonal in transversity

$\hookrightarrow$ consider J_q contribution from quarks of given transversity

$$\left\langle J_{q,s_y}^y \right\rangle = \int d^3x \left\langle T_{q,s_y}^{++} \cdot x \right\rangle$$

- $\perp$ deformation of $\perp$ polarized quark distributions

$\hookrightarrow$ analog to Ji’s sum rule (unpol target) [M.B. hep-ph/0505189]

$$\left\langle J_{q,s_y}^y \right\rangle = \frac{1}{4}\int dx \left[2\tilde{H}_T(x,0,0) + E_T(x,0,0)\right] x$$

“correlation between quark transversity and J_q”

- cannot directly measure $2\tilde{H}_T + E_T$:-P

$\hookrightarrow$ need LGT calcs., or ...

Boer-Mulders function

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x,\mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x,\mathbf{k}_\perp^2) - h_1^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S_q}{M}\right]$$

$\hookrightarrow$ position space asymmetry provides physical mechanism for Boer-Mulders function:

- attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x,\mathbf{k}_\perp)$ and the chirally odd GPD $2\tilde{H}_T + E_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x,\mathbf{k}_\perp)$ and the GPD E.

  - sign of $h_1^\perp$ opposite to sign of $2\tilde{H}_T + E_T$

Boer Mulders Function

Applications of "$\frac{h_1^\perp}{2\tilde{H}_T+E_T} \approx \frac{f_{1T}^\perp}{E}$":

- measure $h_1^\perp \quad \Rightarrow \quad$ sign of $2\tilde{H}_T + E_T$

$\hookrightarrow$ sign of spin-orbit correlation in nucleon wave function

- LGT calcs. of $2\tilde{H}_T + E_T \quad \Rightarrow \quad$ predictions for $h_1^\perp$

Summary

- ↪ knowledge of GPDs for $\xi = 0$ provides novel information about nonperturbative parton structure of nucleons: distribution of partons in $\perp$ plane

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation, e.g. $q(x, \mathbf{b}_\perp) > 0$ for $x > 0$
- $\frac{\Delta_\perp}{2M} E(x, 0, -\mathbf{\Delta}_\perp^2)$ describes how the momentum distribution of unpolarized partons in the $\perp$ plane gets transversely distorted when is nucleon polarized in $\perp$ direction.
- (attractive) final state interaction in semi-inclusive DIS converts $\perp$ position space asymmetry into $\perp$ momentum space asymmetry
- ↪ simple physical explanation for observed Sivers effect in $\gamma^* p \rightarrow \pi X$

Summary

- New “sum rule”: $\left\langle J_q^x(s_q^x)\right\rangle_{unpol.target} = \frac{1}{4}\int dx \left[2\tilde{H}_T + E_T\right] x$
  - $2\tilde{H}_T + E_T$ measures correlation between $\perp$ spin and $\perp$ angular momentum (M.B., hep-ph/0505189)
  - physical explanation for Boer-Mulders effect; relation between $h_1^\perp$ and the GPDs $2\tilde{H}_T + E_T$
- GPDs vs. $q(x, \mathbf{b}_\perp)$: M.B., PRD **62**, 71503 (2000), Int. J. Mod. Phys. **A18**, 173 (2003); see also D. Soper, PRD **15**, 1141 (1977).
- Connection to SSA in M.B., PRD **69**, 057501 (2004); NPA **735**, 185 (2004); PRD **66**, 114005 (2002).