

# Thermalization of Heavy Quarks in the QGP

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April 28, 2005

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# Outline

Motivation

Chiral Heavy-Quark Model

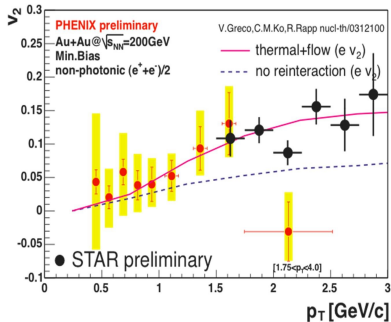
The Fokker-Planck Equation

Friction and Diffusion Coefficients

Relativistic Langevin Process

# Motivation

- ▶  $p_T$  spectra and  $v_2$  of D mesons
- ▶ single-electron  $v_2$  measurements from PHENIX, STAR '04
- ▶ coalescence model describes data under assumption of **flowing thermalized** c quarks



# Motivation

- ▶ importance of dissociation and **regeneration** in



- ▶ in-medium spectral properties of charmonia in QGP (Grandchamp, Rapp '02)
- ▶ importance of **thermalization** of heavy quarks

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- ▶ in-medium spectral properties of charmonia in QGP (Grandchamp, Rapp '02)
- ▶ importance of **thermalization** of heavy quarks
- ▶ Possible mechanism: Survival of “D-mesonic resonances” above  $T_c$
- ▶ suggestive from lattice QCD (Umeda et al '02, Datta et al '03)

## Free Lagrangian: Particle Content

Chiral symmetry  $SU_V(2) \otimes SU_A(2)$  in light-quark sector of QCD

$$\mathcal{L}_D^{(0)} = \sum_{i=1}^2 [(\partial_\mu \Phi_i^\dagger)(\partial^\mu \Phi_i) - m_D^2 \Phi_i^\dagger \Phi_i] + \text{massive (pseudo-)vectors } D^*$$

$\Phi_i$ : two doublets: **pseudo-scalar**  $\sim \begin{pmatrix} \overline{D^0} \\ D^- \end{pmatrix}$  and **scalar**

$\Phi_i^*$ : two doublets: **vector**  $\sim \begin{pmatrix} \overline{D^{0*}} \\ D^{-*} \end{pmatrix}$  and **pseudo-vector**

$$\mathcal{L}_{qc}^{(0)} = \bar{q} i \not{\partial} q + \bar{c} (i \not{\partial} - m_c) c$$

$q$ : light-quark doublet  $\sim \begin{pmatrix} u \\ d \end{pmatrix}$

$c$ : singlet

# Chiral Symmetry

Infinitesimal version:

$$q \rightarrow (1 + i\delta\vec{\phi}_V\vec{t} + i\delta\vec{\phi}_A\vec{t}\gamma_5)q, \quad c \rightarrow c.$$

Light quarks **massless** in chiral limit!

$$\Phi_1 \rightarrow \Phi_1 + i\delta\vec{\phi}_V\vec{t}\Phi_1 + i\delta\vec{\phi}_A\vec{t}\Phi_2,$$

$$\Phi_2 \rightarrow \Phi_2 + i\delta\vec{\phi}_V\vec{t}\Phi_2 + i\delta\vec{\phi}_A\vec{t}\Phi_1.$$

Mesons must have **chiral partners**

In the vacuum: chiral symmetry **spontaneously broken**

In **QGP**: chiral symmetry **restored**

## Interactions

Interactions determined by **chiral** symmetry

Strong interactions also preserve parity

For transversality of vector mesons: use **heavy-quark effective theory vertices**

$$\begin{aligned} \mathcal{L}_{\text{int}} = & -G_S \left( \bar{q} \frac{1 + \not{v}}{2} \Phi_1 c_v + \bar{q} \frac{1 + \not{v}}{2} i\gamma^5 \Phi_2 c_v + h.c. \right) \\ & -G_V \left( \bar{q} \frac{1 + \not{v}}{2} \gamma^\mu \Phi_{1\mu}^* c_v + \bar{q} \frac{1 + \not{v}}{2} i\gamma^\mu \gamma^5 \Phi_{2\mu}^* c_v + h.c. \right) \end{aligned}$$

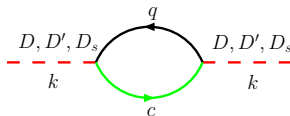
$v$ : four momentum of heavy quark in **HQET**: spin symmetry

$\Rightarrow G_S = G_V$



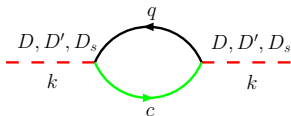
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Dressing the  $D$  mesons with self-energies



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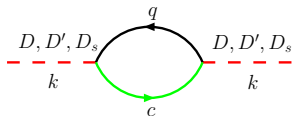


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$$\partial_{k^2} \Pi_D(k^2 = 0) = 0, \quad \Pi_D(k^2 = 0) = 0$$

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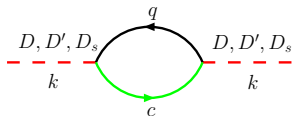
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or **dipole form-factor cutoff**:

$$F_{\text{dip}} = \left( \frac{2\Lambda^2}{2\Lambda^2 + k_{\text{cm}}^2} \right)^2, \quad k_{\text{cm}} = \frac{s - m_c^2}{2\sqrt{s}}$$

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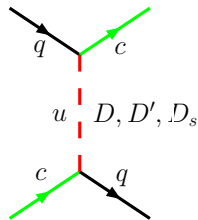
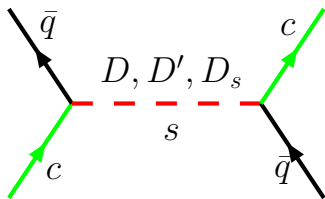
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Bare mass and coupling adjusted such that

$$m_D = 2 \text{ GeV}, \quad \Gamma_D = (0.3 \dots 0.8) \text{ GeV}$$

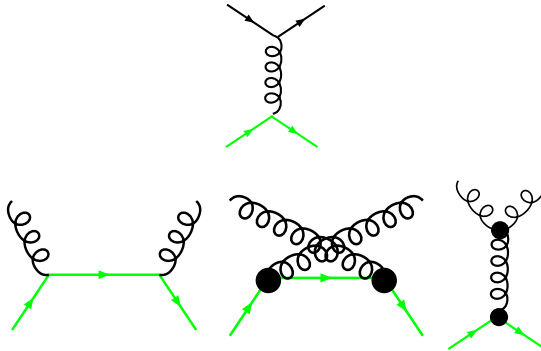
# Resonance Scattering

heavy-light-(anti-)quark scattering



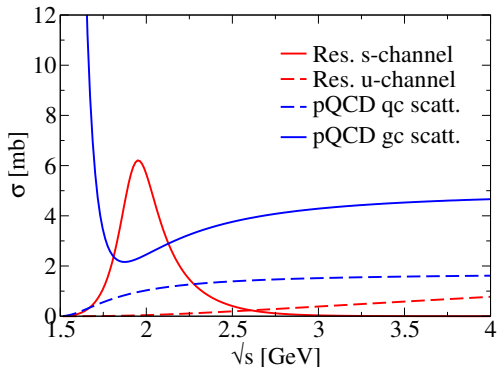
## Contributions from pQCD

Lowest-order matrix elements (Cambridge '79)



In-medium **Debye-screening mass** for  $t$ -channel gluon exchange:  
 $\mu_g = gT$ ,  $\alpha_s = 0.3, 0.4, 0.5$

## Cross sections



- ▶ total pQCD and resonance cross sections: comparable in size
- ▶ BUT pQCD forward peaked  $\leftrightarrow$  resonance isotropic
- ▶ resonance scattering more effective for friction and diffusion

# The Fokker-Planck Equation

heavy particle (**c quarks**) in a **heat bath** of light particles (QGP)

$$\frac{\partial f(t, \vec{p})}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(t, \vec{p}) + \frac{\partial}{\partial p_j} B_{ij}(t, \vec{p}) \right] f(t, \vec{p})$$

Assumption: Relevant scattering processes are **soft**



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$A_i$  and  $B_{ij}$  given by **averages** over initial momenta  $\vec{q}$  of light particles and **summation** over final states (Svetitsky '88):

$$\langle X(\vec{p}') \rangle = \frac{1}{\gamma_c} \frac{1}{2E_p} \int \frac{d^3 \vec{q}}{(2\pi)^3 2E_q} \int \frac{d^3 \vec{q}'}{(2\pi)^3 2E_{q'}} \int \frac{d^3 \vec{p}'}{(2\pi)^3 2E_{p'}} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p + q - p' - q') \hat{f}(\vec{q}) X(\vec{p}')$$

# Friction and Diffusion Coefficients

For  $t, \vec{p}$ -independent coefficients:

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial \vec{p}} (\vec{p} f) + D \frac{\partial^2}{\partial \vec{p}^2} f$$

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Solution for **c quark** with given momentum  $\vec{p}_0$  at  $t = 0$ :

$$G(t, \vec{p} | \vec{p}_0) = \left\{ \frac{\gamma}{2\pi D [1 - \exp(-2\gamma t)]} \right\}^{3/2} \exp \left\{ -\frac{\gamma}{2D} \frac{[\vec{p} - \vec{p}_0 \exp(-\gamma t)]^2}{1 - \exp(-2\gamma t)} \right\}$$

- ▶  $\langle \vec{p} \rangle = \vec{p}_0 \exp(-\gamma t) \Rightarrow \gamma =$  friction coefficient (**dissipation**)
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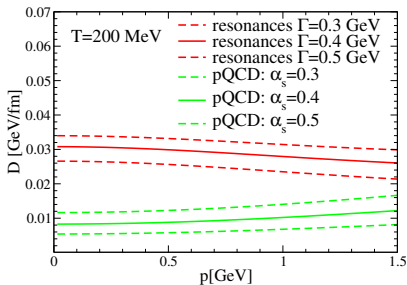
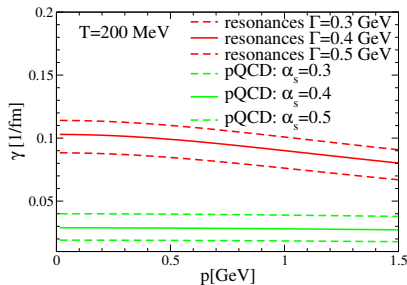
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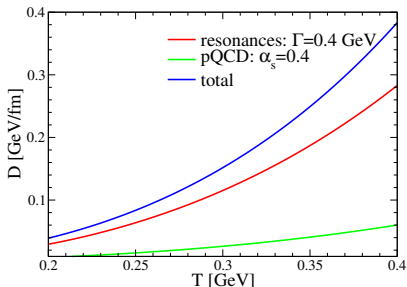
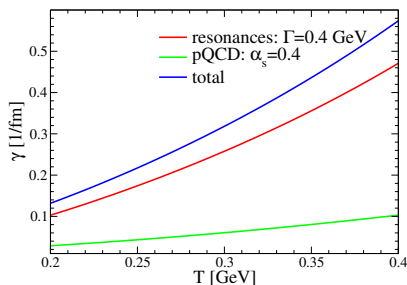
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- ▶  $t \rightarrow \infty$ : temperature  $T^* = D/\gamma m_c$
- ▶ consistency condition  $T^* \stackrel{!}{=} T$  (**heat bath**-temperature)
- ▶ fulfilled within  $\sim 15\%$  for relevant temperature region

# The Coefficients: pQCD vs. resonance scattering



- ▶ only weakly  $p$ -dependent
- ▶ resonance contributions factor  $\sim 2 \dots 3$  higher than pQCD!

## The Coefficients: pQCD vs. resonance scattering



- ▶ temperature dependence  $\Rightarrow$  need to treat Fokker-Planck equation with **time-dependent** coefficients
- ▶ Solvable with method of characteristics

## Time evolution of the fire ball

- ▶ Simple **fire-ball** parameterization:

$$V(\tau) = \pi(z_0 + v_z \tau) \left( r_0 + \frac{1}{2} a_{\perp} \tau^2 \right)^2$$

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- ▶ **Equation of state**:

$$s = \frac{4\pi^2}{90} T^3 (16 + 10.5 n_f^*), \quad n_f^* = 2.5$$

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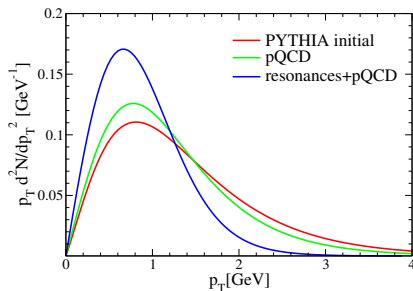
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- ▶ **initial condition** from PYTHIA

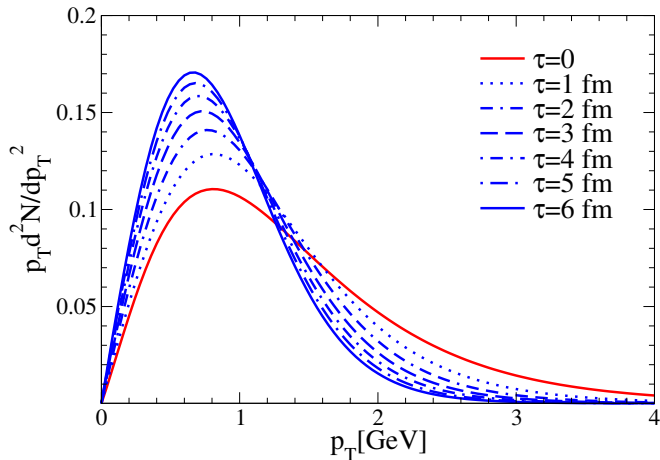
$$\frac{d^2 N}{dp_T^2} := f(p_T, t=0) \propto \frac{(p_T + A)^2}{(1 + p_T/B)^\alpha}$$

## Evolved $p_T$ spectra



- ▶ initially  $\sqrt{\langle p_T^2 \rangle} = 1.66$  GeV
- ▶ with pQCD: not much change in spectrum
- ▶ with resonance contributions:  $p_T^{(\max)} \sim 0.66$  GeV
- ▶ nearly thermal:  $T \sim 290$  MeV

## Evolution of $p_T$ spectra



# Relativistic Langevin Process

- ▶ aim: include (asymmetric) flow +  $p$ -dep. FP coefficients
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- ▶ in (local) restframe of the heat bath

$$\delta\vec{x} = \frac{\vec{p}}{E}\delta t$$

$$\delta\vec{p} = -\gamma(t, \vec{p} + \delta\vec{p})\vec{p}\delta t + \delta W(t, \vec{p} + \delta\vec{p})$$

$$P(\delta\vec{W}) \propto \exp\left[-\frac{1}{4D(t, \vec{p} + \delta\vec{p})\delta t E^2/m^2}\right]$$

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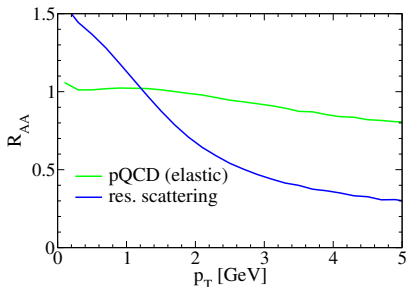
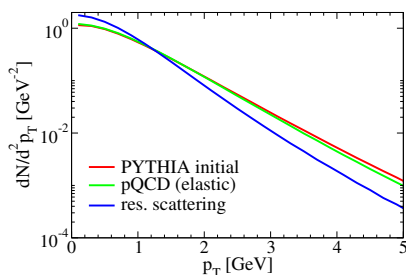
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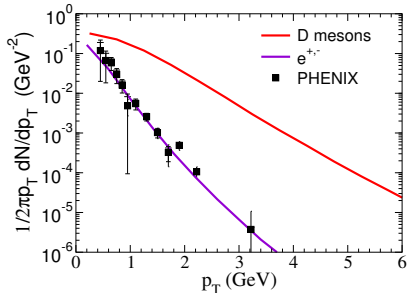
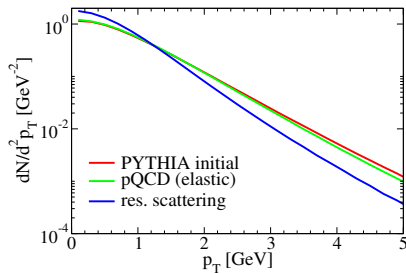
- ▶ **Hänggi-Klimontovic** realization  $\Leftrightarrow$  rel. Maxwell distribution as equilibrium limit with unchanged FP coefficients

## Observables: $p_T$ -spectra, $v_2$ , ( $R_{AA}$ )

- ▶ use elliptic flow parametrization for fireball, based on hydro [Kolb et al]
- ▶ boost “labframe”  $\leftrightarrow$  local heat-bath-rest frame (see also [Moore, Teaney 2004])

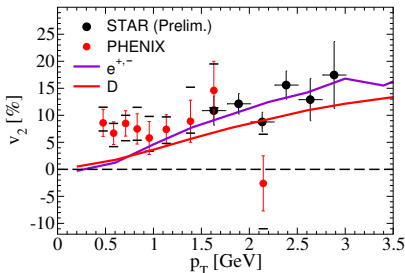
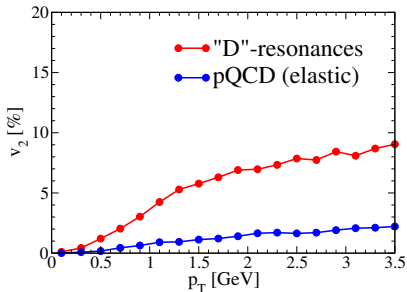


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- ▶  $D$ -meson  $v_2$  via coalescence
- ▶  $e^{\pm}$  from subsequent decay

## Conclusions

- ▶ Assumption: survival of **resonances** in the QGP
- ▶ possible mechanism for **strong interactions** beyond  $T_c$
- ▶ **Equilibration** of heavy quarks in QGP
- ▶ **Observables** via Langevin approach and coalescence