J/Ψ AND Ψ' POLARIZATION IN POLARIZED pp COLLISIONS at the RHIC

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INTRODUCTION

J/Ψ PRODUCTION MECHANISM AT FIXED TARGET AND COLLIDER EXPERIMENTS

J/Ψ PRODUCTION MECHANISM IN UNPOLARIZED pp COLLISIONS AT RHIC

J/Ψ PRODUCTION IN POLARIZED pp COLLISIONS AT RHIC

CONCLUSIONS

INTRODUCTION

J/Ψ at RHIC: (Various Aspects)

1) Unpolarized J/Ψ In Unpolarized pp, dAu, AuAu Collisions

2) J/Ψ Polarization In Unpolarized pp, and AuAu Collisions

3) Unpolarized J/Ψ In Polarized pp Collisions

4) J/Ψ Polarization In Polarized pp Collisions

5) J/Ψ Production/Suppression in Quark-Gluon Plasma

In AuAu Collisions at RHIC:

J/Ψ Suppression is Proposed to be a Signature of Quark-Gluon Plasma Detection

In Polarized pp Collisions at RHIC:

J/Ψ Production Can be Used to Extract Polarized Gluon Distribution Function Inside the Proton

J/Ψ Polarization at RHIC:

As J/Ψ Polarization at Tevatron is not Explained by Theory, It might be Useful to Look at the Same at RHIC

J/Ψ Polarization in Polarized pp at RHIC:

This is Unique Measurement (Not available at any other Collider), Will Test Spin Transfer Process in pQCD

J/Ψ Production In Unpolarized pp Collisions at RHIC:

First, One should Understand the J/Ψ Production Mechanism in Unpolarized pp Collisions at RHIC Before Going to AuAu Collisions or Polarized pp Collisions

Different Charm Quarkonium States:

Quarkonium: Masses	States ($^{2S+1}L_J$):	
J/Ψ	$^3S_1$	**3.097 (GeV)**
χ_0	$^3P_0$	**3.414 (GeV)**
χ_1	$^3P_1$	**3.507 (GeV)**
χ_2	$^3P_2$	**3.551 (GeV)**
η_c	$^1S_0$	**2.983 (GeV)**

$\chi' s$ **Decay to** J/Ψ **via:** $\quad ^3P_J \to {}^3S_1\gamma$

Branching Ratios

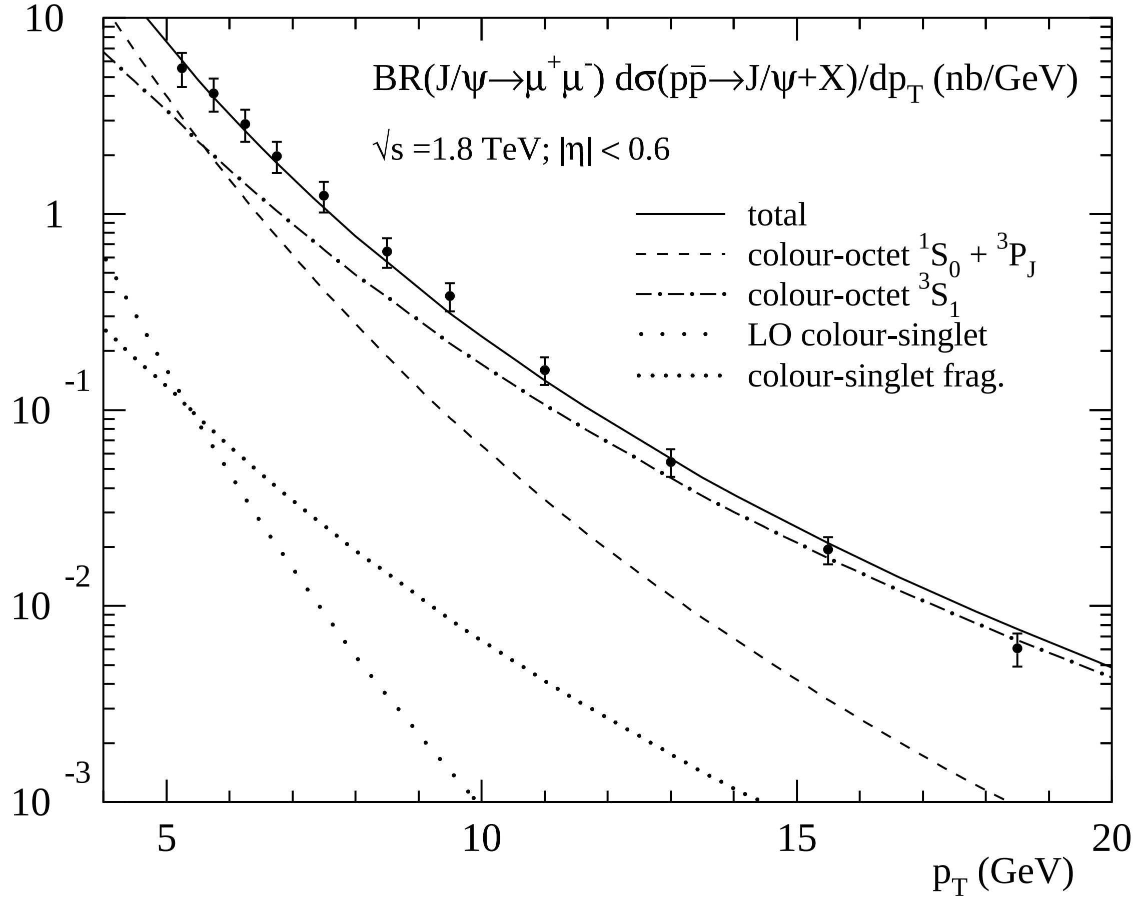
$\chi_0 \to J/\Psi\gamma$	**0.027**
$\chi_1 \to J/\Psi\gamma$	**0.315**
$\chi_2 \to J/\Psi\gamma$	**0.154**

Heavy Quarkonium Production Mechanisms

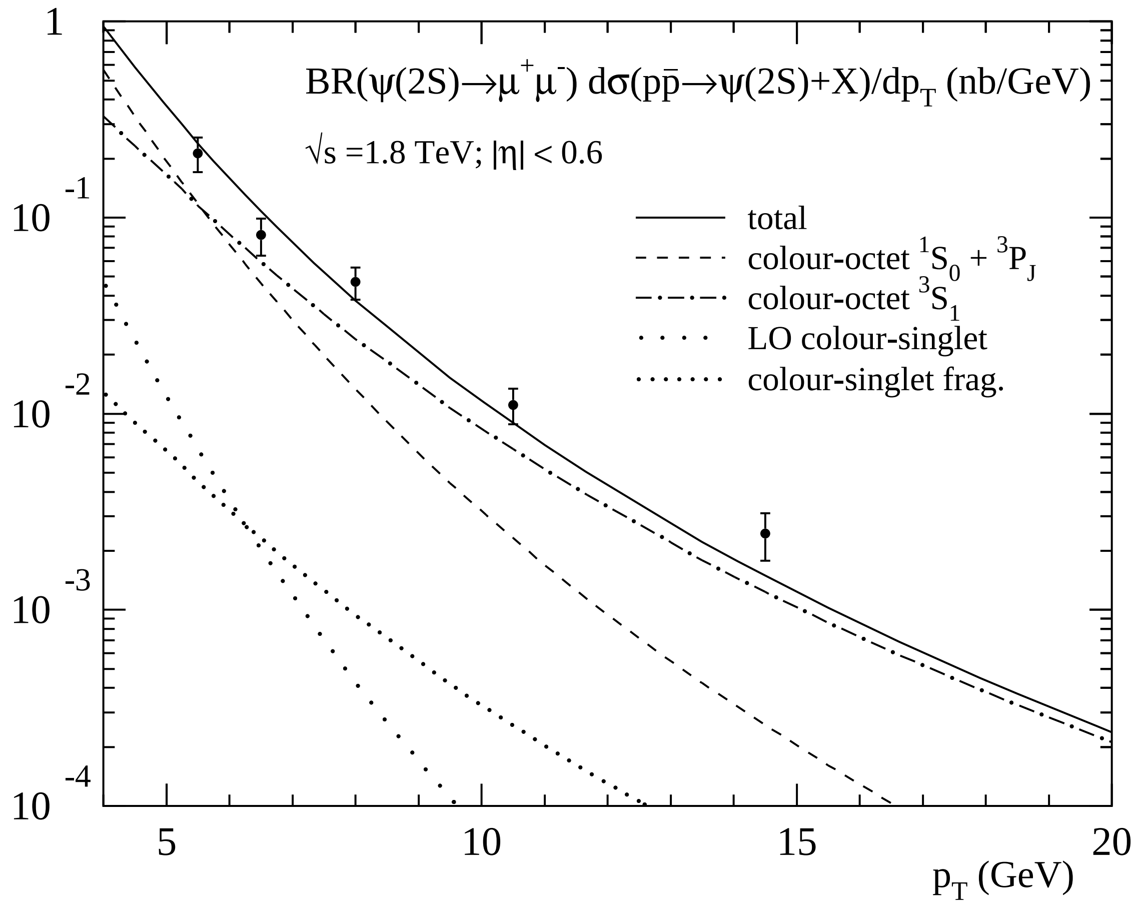
1) Color Singlet Mechanism
2) Color Octet Mechanism

Color Singlet Mechanism:
The $c\bar{c}$ is Formed in a Color Singlet State with Same Quantum Numbers (L,S,J) of the Charmonium

Non-Perturbative Matrix Elements Can be Obtained From Solving Non-Relativistic Schrodinger Equations or Can Be Taken From Experiments



Failure of Color Singlet Mechanism at Tevatron



Failure of Color Singlet Mechanism at Tevatron

Color Octet Mechanism
(Bodwin, Braaten, Lepage)

Two Parameters:

Coupling Constant: $\alpha_s(M)$**:**

Relative Velocity in $Q\bar{Q}$ **Bound State:** v**:**

$\alpha_s(2M_c) \sim 0.24$

$\alpha_s(2M_b) \sim 0.18$

$v^2 \sim 0.23$ **For** $C\bar{C}$ **System**

$v^2 \sim 0.08$ **For** $B\bar{B}$ **System**

Different Energy Scales: (In Terms of v):

1) M: Quark Mass: Scale For Annihillation Decays.

2) Mv: Momentum: Length Scale For the Size of the Quarkonium States.

3) Mv^2: Kinetic Energy: Scale of Splitting Between Different Excitation Sates (Both Radial and Angular)

$M >> Mv >> Mv^2$

NRQCD LAGRANGIAN DENSITY

$$\mathcal{L}_{NRQCD} = \mathcal{L}_{light} + \mathcal{L}_{heavy} + \mathcal{L}_{correction}$$

$$\mathcal{L}_{light} = -\frac{1}{2} F^{a\mu\nu} F^{a}_{\mu\nu} + \Sigma\, \bar{q} \gamma_\mu D^\mu [A] q$$

Two component Dirac Spinor

$$\Psi_{heavy} \equiv \begin{matrix} \psi \\ \chi \end{matrix}$$

At Leading Order

$$\mathcal{L}_{heavy} = \psi^\dagger (iD_t + \frac{D^2}{2M}) \psi + \chi^\dagger (iD_t + \frac{D^2}{2M}) \chi$$

Higher Order Corrections

$$\mathcal{L}_{correction} = \frac{1}{8M^3}[\psi^\dagger D^4\psi - \chi^\dagger D^4\chi]$$
$$+ \frac{g}{8M^2}[\psi^\dagger(D\cdot E - E\cdot D)\psi + \chi^\dagger(D\cdot E - E\cdot D)\chi]$$
$$+ \frac{ig}{8M^2}[\psi^\dagger\sigma\cdot(D\times E - E\times D)\psi + \chi^\dagger\sigma\cdot(D\times E - E\times D)\chi]$$
$$+ \frac{g}{2M}[\psi^\dagger\sigma\cdot B\psi - \chi^\dagger\sigma\cdot B\chi]$$

Heavy Quarkonium Production Amplitude is:

$$|\psi_Q >= O(1)|Q\bar{Q}[^3S_1^{(1)}] > +O(v)|Q\bar{Q}[^3P_J^{(8)}]g > + O(v^2)|Q\bar{Q}[^3S_1^{(1,8)}]gg > +O(v^2)|Q\bar{Q}[^1S_0^{(8)}]g > + O(v^2)|Q\bar{Q}[^3D_J^{(1,8)}]gg > +.....$$

and the wave functions of P-wave ortho-quarkonium state $|\chi_{QJ}>$:

$$|\chi_{QJ} >= O(1)|Q\bar{Q}[^3P_J^{(1)}] > +O(v)|Q\bar{Q}[^3S_1^{(8)}]g > +....$$

Factorization In Heavy Quarkonium Production (Fragmentation Processes)

(G. C. Nayak, J-W Qiu and G. Sterman, Phys. Lett. B613 (2005) 45; hep-ph/0509021).

- **Soft gluons in heavy quarkonium production at high p_T**
- **Uncancelled infrared poles at NNLO** *not* **matched by conventional NRQCD matrix elements**
- **NNLO Fix:** **Gauge invariance $\Rightarrow$ Modification of NRQCD operators**
- **Nonabelian phases: Wilson lines**
- **NRQCD Heavy Quarkonium production:**

$$d\sigma_{A+B\to H+X}(p_T) = \sum_n d\hat{\sigma}_{A+B\to c\bar{c}[n]+X}(p_T)\,\langle \mathcal{O}_n^H \rangle$$

- **With Vevs of the "production" operators**

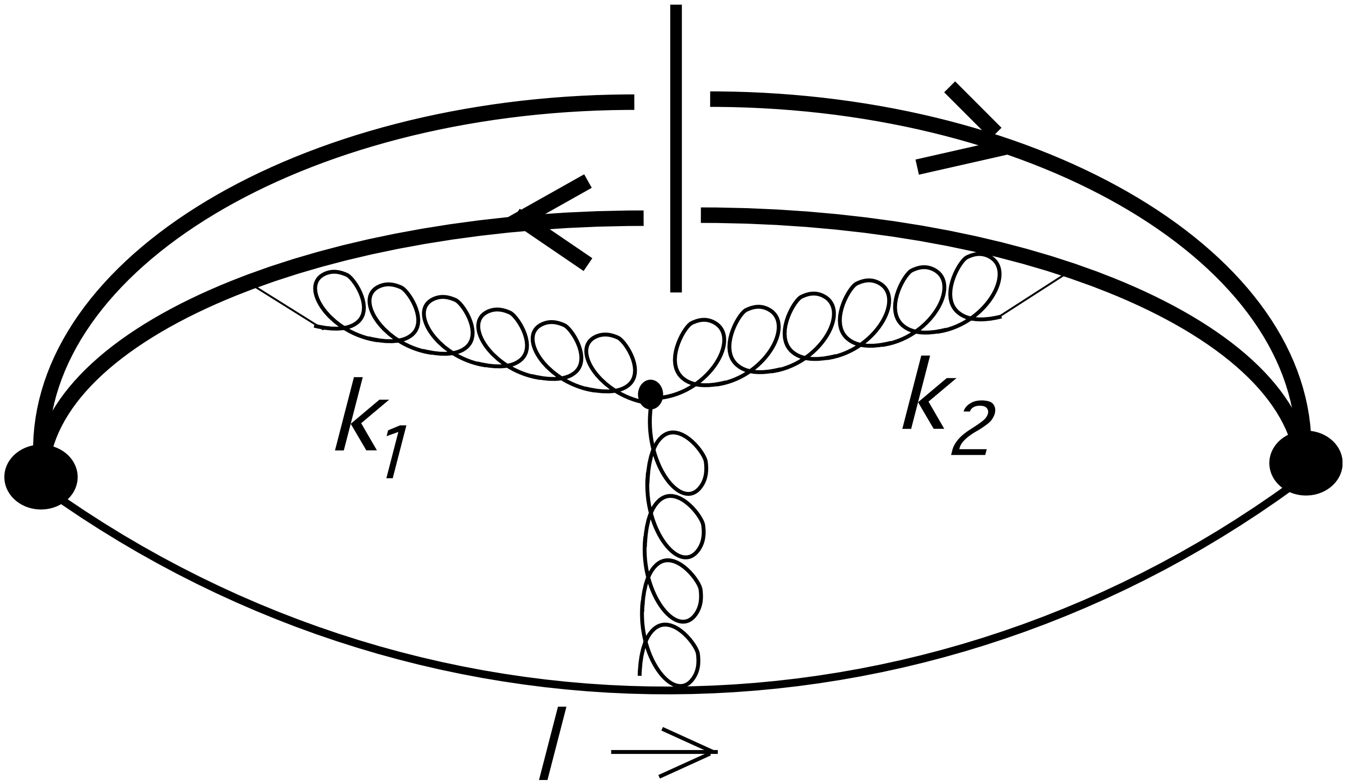
$$\mathcal{O}_n^H(0) = \chi^\dagger \mathcal{K}_n \psi(0)\,\left(a_H^\dagger a_H\right)\,\psi^\dagger \mathcal{K}_n' \chi(0)$$

$$\mathcal{O}_n^H(0) = \chi^\dagger \mathcal{K}_n \psi(0)\,\left(a_H^\dagger a_H\right)\,\psi^\dagger \mathcal{K}_n' \chi(0)\,, \qquad (1)$$

- **Only color-singlet $\mathcal{K}$'s give gauge invariant $\mathcal{O}$'s**

  NNLO in fragmentation: uncancelled IR divergences

- **Generalization to NNLO**
- **The diagrams that *don't cancel* from:**



(c)

NNLO Two Loop Diagram With Massive Quarks and 3 Gluon Vertex

- **The IR divergent expression to order $q^2 \propto v^2$:**

$$\Sigma^{(2c)}(P,q,l) = -16i\, g^4\mu^{4\varepsilon}\int \frac{d^Dk_1}{(2\pi)^D}\frac{d^Dk_2}{(2\pi)^D}, 2\pi\delta(k_1^2) l^\lambda\, V_{\nu\mu\lambda}[k_1,k_2]$$
$$\times\ [q^\mu(P\cdot k_1)-(q\cdot k_1)P^\mu]\ [q^\nu(P\cdot k_1)-(q\cdot k_2)P^\nu]$$
$$\times\frac{1}{[P\cdot k_1+i\epsilon]^2\ [P\cdot k_2-i\epsilon]^2}$$
$$\times\frac{1}{[k_2^2-i\epsilon]\ [(k_2-k_1)^2-i\epsilon]\ [l\cdot(k_1-k_2)-i\epsilon]},$$

- $V_{\nu\mu\lambda}[k_1,k_2]$**: momentum part of three-gluon coupling.**

- **And the result is:**

$$\Sigma^{(2)}(P,q,l) = \alpha_s^2\,\frac{4}{3\varepsilon}\,[\frac{(P\cdot q)^2}{P^4} - \frac{q^2}{P^2}].$$

- **In rest frame:**

$$\Sigma(P,q,l) = \alpha_s^2\,\frac{4}{3\varepsilon}\,\frac{\vec{q}^{\,2}}{4m_c^2} = \alpha_s^2\,\frac{1}{3\varepsilon}\,\frac{\vec{v}^{\,2}}{4}, \qquad (2)$$

- **Breakdown of the simplest topological factorization**

  of infrared divergences at NNLO

- **Conclude: we need the Wilson lines**

  Redefinition NRQCD Matrix Elements

- **Resolution: as for fragmentation, supplement fields by Wilson lines:**

  $$\Phi_l[x,A] = \exp\left[-ig\int_0^\infty d\lambda\, l\cdot A(x+\lambda l)\right]$$

- **Our new, gauge-invariant NRQCD operators:**

$$\mathcal{O}_n^H(0) \to \chi^\dagger \mathcal{K}_{n,c}\psi(0)\, \Phi_l^\dagger[0,A]_{cb} \left(a_H^\dagger a_H\right)\, \Phi_l[0,A]_{ba}\, \chi^\dagger \mathcal{K}'_{n,a}\psi(0)\,,$$

J/Ψ Polarization in Polarized pp Collisions at RHIC

Leading Order Calculations:

$$M_{q\bar{q}\to Q\bar{Q}} \;=\; \tfrac{g^2}{P^2}\,\bar{v}(k_2)\gamma_\mu T^a u(k_1)\,\bar{u}(p_1)\gamma^\mu T^a v(p_2)$$

$P^\mu \;=\; p_1^\mu + p_2^\mu \;=\; k_1^\mu + k_2^\mu$; $\quad p_1^\mu = P^\mu/2 + L_j^\mu q^j$; $p_2^\mu = P^\mu/2 - L_j^\mu q^j$.

L_j^μ **is The Boost Matrix:** $\quad L_j^0 = \frac{P^j}{2E_q}$
$L_j^i = \delta^{ij} + \frac{P^i P^j}{\vec{P}^2}[\frac{P^0}{2E_q} - 1]$

In Terms of Two Component Dirac Spinors (η, ξ):

$$|M_{q\bar{q}\to Q\bar{Q}}|^2 = \frac{g^4}{4m^2}\, \eta'^\dagger \sigma^i T^a \xi' \, L_i^\mu \, \bar{u}(k_1)\gamma_\mu T^a$$
$$v(k_2)\, \bar{v}(k_2)\gamma_\nu T^b u(k_1)\, L_j^\nu \, \xi^\dagger \sigma^j T^b \eta$$

$$u(k_1)\bar{u}(k_1) = \frac{1}{2}\,(1+\lambda_1\gamma_5)\,\gamma_\mu k_1^\mu$$
$$v(k_2)\bar{v}(k_2) = \frac{1}{2}\,(1-\lambda_2\gamma_5)\,\gamma_\mu k_2^\mu$$

$$\Delta\, |M_{q\bar{q}\to Q\bar{Q}}|^2 = \frac{g^4}{4m^2}\, \eta'^\dagger \sigma^i T^a \xi' \, \xi^\dagger \sigma^j T^a \eta$$
$$[2m^2 n_i n_j - \delta_{ij}(k_1 \cdot k_2)]$$

$(k_2 \cdot L)_i = -(k_1 \cdot L)_i = m\ n_i$. **Here** $n_i(n_j)$ **are the unit three-vectors which specify the polarizations of the heavy quarks (antiquarks) in the charmonium bound state.**

Hence For Leading Order $q\bar{q}$ Fusion Process We Get

$$\Delta \ |M_{q\bar{q}\to Q\bar{Q}}|^2 \ = \ \frac{g^4}{4} \ [n_i n_j - \delta_{ij}] \ \eta'^{\dagger}\sigma^i T^a \xi' \, \xi^{\dagger}\sigma^j T^a \eta$$

$$\Delta \ |M_{q\bar{q}\to Q\bar{Q}}|^2 \ = \ \frac{4\pi^2\alpha_s^2}{9} \ [n_i n_j - \delta_{ij}] \ \eta'^{\dagger}\sigma^i T^a \xi' \xi^{\dagger}\sigma^j T^a \eta$$

Non-Perturbative Matrix Element is:

$$4m^2\eta'^{\dagger}\sigma^i T^a \xi' \xi^{\dagger}\sigma^j T^a \eta \equiv < \chi^{\dagger}\sigma^i T^a \psi P_{H(\lambda)} \psi^{\dagger}\sigma^j T^a \chi >= \frac{4}{3} U^{\dagger}_{\lambda i} U_{j\lambda} m < \mathcal{O}_8^H(^3S_1) >$$

Where The Orthogonal Matrix $U_{\lambda i}$ Relates the Helicity States in Spherical Basis State λ with The Cartesian Basis State i.

$$\Sigma_i U_{\lambda i} U^{\dagger}_{i\lambda} = 1; \qquad \Sigma_i U_{\lambda i} n^i = \delta_{\lambda 0}$$

$$4m^2 \;\; \eta'^\dagger \xi' \, \xi^\dagger \eta \;\; \equiv \;\; < \chi^\dagger \psi \, P_{H(\lambda)} \;\; \psi^\dagger \chi > \;\; = \;\; \frac{4}{3} m < \mathcal{O}_1^H({}^1S_0) > ,$$

$$4m^2 \;\; \eta'^\dagger T^a \xi' \, \xi^\dagger T^a \eta \;\; \equiv \;\; < \chi^\dagger T^a \psi \, P_{H(\lambda)} \;\; \psi^\dagger T^a \chi > \;\; = \;\; \frac{4}{3} m < \mathcal{O}_8^H({}^1S_0) > ,$$

$$4m^2 \;\; \eta'^\dagger \sigma^i \xi' \, \xi^\dagger \sigma^j \eta \;\; \equiv \;\; < \chi^\dagger \sigma^i \psi \, P_{H(\lambda)} \;\; \psi^\dagger \sigma^j \chi > \;\; = \;\; \frac{4}{3} U_{\lambda i}^\dagger U_{j\lambda} m < \mathcal{O}_1^H({}^3S_1) > ,$$

$$4m^2 \;\; \eta'^\dagger \sigma^i T^a \xi' \, \xi^\dagger \sigma^j T^a \eta \;\; \equiv \;\; < \chi^\dagger \sigma^i T^a \psi \, P_{H(\lambda)} \;\; \psi^\dagger \sigma^j T^a \chi > \;\; = \;\; \frac{4}{3} U_{\lambda i}^\dagger U_{j\lambda} m < \mathcal{O}_8^H({}^3S_1) > ,$$

$$4m^2 \;\; q^n q^m \eta'^\dagger \sigma^i \xi' \, \xi^\dagger \sigma^j \eta \;\; \equiv \;\; < \chi^\dagger (-\tfrac{i}{2} D^m) \sigma^i \psi \, P_{H(\lambda)} \;\; \psi^\dagger (-\tfrac{i}{2} D^n) \sigma^j \chi >$$

$$= \;\; 4 U_{\lambda i}^\dagger U_{j\lambda} \delta^{mn} m < \mathcal{O}_1^H({}^3P_0) > ,$$

$$4m^2 \;\; q^n q^m \eta'^\dagger \sigma^i T^a \xi' \xi^\dagger \sigma^j T^a \eta \;\; \equiv \;\; < \chi^\dagger (-\tfrac{i}{2} D^m) \sigma^i T^a \psi \, P_{H(\lambda)} \;\; \psi^\dagger (-\tfrac{i}{2} D^n) \sigma^j T^a \chi >$$

$$= \;\; 4 U_{\lambda i}^\dagger U_{j\lambda} \delta^{mn} m < \mathcal{O}_8^H({}^3P_0) >$$

Hence the Matrix Element Square is:

$$\Delta|M_{q\bar{q}\to H(\lambda)}|^2 \;=\; -\frac{4\pi^2\alpha_s^2}{27}\,[1-\delta_{\lambda 0}]\;<\mathcal{O}_8^H({}^3S_1)>$$

The Partonic Level Cross Section is:

$$\Delta\sigma_{q\bar{q}\to H(\lambda)} = -\delta(\hat{s}-4m^2)\frac{\pi^3\alpha_s^2}{27m^3}[1-\delta_{\lambda 0}] < \mathcal{O}_8^H({}^3S_1) >$$

Consider Gluon-Gluon Fusion Process at LO:

Adding $\hat{s}$, $\hat{t}$, and $\hat{u}$ Channel Feynman Diagrams

$M_{gg\to Q\bar{Q}} = -g^2 \;\; \epsilon^a_\mu(k_1)\epsilon^{*b}_\nu(k_2)[(\frac{1}{6}\delta^{ab} + \frac{1}{2}d^{abc}T^c)S^{\mu\nu} + \frac{i}{2}f^{abc}T^cF^{\mu\nu}]$

$S^{\mu\nu} = \bar{u}(p_1)[\frac{\gamma^\mu(\not{p}_1-\not{k}_1+m)\gamma^\nu}{2p_1\cdot k_1} + \frac{\gamma^\nu(\not{p}_1-\not{k}_2+m)\gamma^\mu}{2p_1\cdot k_2}]v(p_2)$

$F^{\mu\nu} = \bar{u}(p_1)[\frac{\gamma^\mu(\not{p}_1-\not{k}_1+m)\gamma^\nu}{2p_1\cdot k_1} - \frac{\gamma^\nu(\not{p}_1-\not{k}_2+m)\gamma^\mu}{2p_1\cdot k_2} + \frac{2}{P^2}V^{\mu\nu\lambda}(k_1,k_2,-k_1-k_2)\gamma_\lambda]v(p_2)$

Simplifying Three Gammma Matrices terms We Get:

$\bar{u}(p_1)[\frac{\gamma^\mu \not{k}_1 \gamma^\nu}{2p_1 \cdot k_1} + \frac{\gamma^\nu \not{k}_2 \ \gamma^\mu}{2p_1 \cdot k_2}]v(p_2) = \frac{i}{2m^2}(k_1 - k_2)_\lambda \epsilon^{\rho\mu\nu\lambda} P_\rho \xi^\dagger \eta$
$+ \frac{(L \cdot k_1)_n}{m^3}[P^\nu L^\mu_j - P^\mu L^\nu_j + 2g^{\mu\nu}(L \cdot k_1)_j - (k_1 - k_2)^\mu L^\nu_j - (k_1 - k_2)^\nu L^\mu_j] q^n \xi^\dagger \sigma^j \eta$
$+ \frac{(L \cdot k_1)_j}{m^3}[P^\mu L^\nu_n - P^\nu L^\mu_n] q^n \xi^\dagger \sigma^j \eta + \frac{1}{m}[P^\mu L^\nu_j - P^\nu L^\mu_j] \xi^\dagger \sigma^j \eta$

$\bar{u}(p_1)[\frac{\gamma^\mu \not{k}_1 \gamma^\nu}{2p_1 \cdot k_1} - \frac{\gamma^\nu \not{k}_2 \gamma^\mu}{2p_1 \cdot k_2}]v(p_2) = \frac{(L \cdot k_1)_n}{2m^4}(k_1 - k_2)_\lambda \epsilon^{\rho\mu\nu\lambda} P_\rho q^n \xi^\dagger \eta$
$- \frac{(L \cdot k_1)_n}{m^3}[P^\nu L^\mu_j + P^\mu L^\nu_j] q^n \xi^\dagger \sigma^j \eta$
$- \frac{1}{m}[2g^{\mu\nu}(L \cdot k_1)_j - (k_1 - k_2)^\mu L^\nu_j - (k_1 - k_2)^\nu L^\mu_j] \ \xi^\dagger \sigma^j \eta$
$+ \frac{2}{m}[L^\mu_n L^\nu_j - L^\nu_n L^\mu_j] q^n \xi^\dagger \sigma^j \eta$

Hence

$$S^{\mu\nu} = \frac{i}{2m^2}(k_1 - k_2)_\lambda \epsilon^{\rho\mu\nu\lambda} P_\rho \xi^\dagger \eta + [\frac{(L\cdot k_1)_j}{m^3}(P^\nu L_n^\mu - P^\mu L_n^\nu - 2g^{\mu\nu}(L\cdot k_1)_n)$$
$$+ \frac{2}{m}[L_n^\mu L_j^\nu + L_n^\nu L_j^\mu] + \frac{1}{m^3}(L\cdot k_1)_n[(k_1-k_2)^\mu L_j^\nu + (k_1 - k_2)^\nu L_j^\mu]]q^n \xi^\dagger \sigma^j \eta$$

$$F^{\mu\nu} = \frac{i(L\cdot k_1)_n}{2m^4}(k_1 - k_2)_\lambda \epsilon^{\rho\mu\nu\lambda} P_\rho q^n \xi^\dagger \eta + [k_2^\nu L_j^\mu - k_1^\mu L_j^\nu]\xi^\dagger \sigma^j \eta$$

Gluon Polarizations Are Given By:

$$\epsilon_\mu^a(k_1, \lambda_1)\epsilon_\nu^{*b}(k_1, \lambda_1) = \frac{1}{2}\delta^{ab}[-g_{\mu\nu} + \frac{k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu}}{k_1\cdot k_2} - i\lambda_1 \ \epsilon_{\mu\nu\rho\delta}\frac{k_1^\rho k_2^\delta}{k_1\cdot k_2}]$$

Using The Relation

$$\epsilon_{\mu\mu'\alpha\beta}k_1^{\alpha}k_2^{\beta} \ = 2m^2\epsilon^{ijk}n_k L_i^{\mu}L_j^{\nu}$$

We Get:

$$\Delta|M_{gg\to Q\bar{Q}}|^2 = -\frac{g^4}{4}\epsilon^{pqr}\epsilon^{p'q'r'}n_r n_{r'}[S^{ab}S^{*ab}L_{\mu p}L_{\nu p'}S^{\mu\nu}L_{\mu' q}L_{\nu' q'}$$
$$S^{*\mu'\nu'} + F^{ab}F^{*ab}L_{\mu p}L_{\nu p'}F^{\mu\nu}L_{\mu' q}L_{\nu' q'}F^{*\mu'\nu'}]$$

Where:

$S^{ab} = \frac{1}{6}\delta^{ab} + \frac{1}{2}d^{abc}T^c$ and $F^{ab} = \frac{i}{2}f^{abc}T^c$

The Cross Terms Vanish Because:

$$S^{ab}F^{*ab} = \ 0 = S^{*ab}F^{ab}$$

The Matrix Element Square for the gg Fusion Process at LO Is:

$$\Delta|M_{gg\to Q\bar{Q}}|^2 = -\frac{\pi^2\alpha_s^2}{9}[\eta'^\dagger\xi'\xi^\dagger\eta + \frac{1}{m^2}[(n\cdot q)n_j q'_{j'} + (n\cdot q')n_{j'}q_j - \frac{3}{2}(n\cdot q)(n\cdot q')n_j n_{j'} - (n\times q')_j(n\times q)_{j'}]\eta'^\dagger\sigma^{j'}\xi'\xi^\dagger\sigma^j\eta + \frac{15}{8}\eta'^\dagger T^a\xi'\xi^\dagger T^a\eta + \frac{15}{8m^2}[(n\cdot q)n_j q'_{j'} + (n\cdot q')n_{j'}q_j - \frac{3}{2}(n\cdot q)(n\cdot q')n_j n_{j'} - (n\times q')_j(n\times q)_{j'}]\eta'^\dagger\sigma^{j'}T^a\xi'\xi^\dagger\sigma^j T^a\eta + \frac{27}{8m^2}(n\cdot q)(n\cdot q')\eta'^\dagger T^a\xi'\xi^\dagger T^a\eta]$$

Different Non-Perturbative Matrix Elements are:

$$4m^2\eta'^\dagger\xi'\xi^\dagger\eta \equiv < \chi^\dagger\psi P_{H(\lambda)}\psi^\dagger\chi > =$$
$$\frac{4}{3}m < \mathcal{O}_1^H(^1S_0) >,$$
$$4m^2\eta'^\dagger T^a\xi'\xi^\dagger T^a\eta \equiv < \chi^\dagger T^a\psi P_{H(\lambda)}\psi^\dagger T^a\chi > =$$
$$\frac{4}{3}m < \mathcal{O}_8^H(^1S_0) >,$$
$$4m^2\eta'^\dagger\sigma^i\xi'\xi^\dagger\sigma^j\eta \equiv < \chi^\dagger\sigma^i\psi P_{H(\lambda)}\psi^\dagger\sigma^j\chi > =$$
$$\frac{4}{3}U^\dagger_{\lambda i}U_{j\lambda}m < \mathcal{O}_1^H(^3S_1) >,$$
$$4m^2q^nq^m\eta'^\dagger\sigma^i\xi'\xi^\dagger\sigma^j\eta \equiv$$
$$< \chi^\dagger(-\frac{i}{2}D^m)\sigma^i\psi P_{H(\lambda)}\psi^\dagger(-\frac{i}{2}D^n)\sigma^j\chi > =$$
$$4U^\dagger_{\lambda i}U_{j\lambda}\delta^{mn}m < \mathcal{O}_1^H(^3P_0) >,$$
$$4m^2q^nq^m\eta'^\dagger\sigma^iT^a\xi'\xi^\dagger\sigma^jT^a\eta \equiv$$
$$< \chi^\dagger(-\frac{i}{2}D^m)\sigma^iT^a\psi P_{H(\lambda)}\psi^\dagger(-\frac{i}{2}D^n)\sigma^jT^a\chi > =$$
$$4U^\dagger_{\lambda i}U_{j\lambda}\delta^{mn}m < \mathcal{O}_8^H(^3P_0) >$$

The Matrix Element Square For The Quarkonium Production With Polarization λ in gg Fusion Process is Given By:

$$\Delta|M_{gg\to H(\lambda)}|^2 = -\frac{\pi^2\alpha_s^2}{27}[< \mathcal{O}_1^H(^1S_0) > +\frac{15}{8} < \mathcal{O}_8^H(^1S_0) >$$
$$+ \frac{3}{m^2}(\frac{1}{2}\delta_{\lambda 0} - 1)[< \mathcal{O}_1^H(^3P_0) > +\frac{15}{8} < \mathcal{O}_8^H(^3P_0) >] + \frac{81}{8m^2} < \mathcal{O}_8^H(^1P_1) >]$$

The Partonic Level Scattering Cross Section is Given By

$$\Delta\sigma_{gg\to H(\lambda)} = -\delta(\hat{s} - 4m^2)\ \frac{\pi^3\alpha_s^2}{108m^3}[< \mathcal{O}_1^H(^1S_0) > +\frac{15}{8} < \mathcal{O}_8^H(^1S_0) >$$
$$+ \frac{3}{m^2}(\frac{1}{2}\delta_{\lambda 0} - 1)[< \mathcal{O}_1^H(^3P_0) > +\frac{15}{8} < \mathcal{O}_8^H(^3P_0) >] + \frac{81}{8m^2} < \mathcal{O}_8^H(^1P_1) >]$$

The J/Ψ Production Cross Section with Polarization λ in Polarized pp Collisions is:

$$\Delta\sigma_{(pp\to J/\psi(\lambda)(\psi'(\lambda)))} = \frac{\pi^3\alpha_s^2}{27sm^3}\int_{4m^2/s}^{1}\frac{dx_1}{x_1}[\Delta f_q(x_1,2m)\,\Delta f_{\bar{q}}(\frac{4m^2}{x_1 s},2m)\,(\delta_{\lambda 0}-1) < \mathcal{O}_8^{J/\psi(\psi')}(^3S_1) > +\frac{15}{32}\,\Delta f_g(x_1,2m)\,\Delta f_g(\frac{4m^2}{x_1 s},2m)\times[\frac{9}{m^2}(1-\frac{1}{2}\delta_{\lambda 0}) < \mathcal{O}_8^{J/\psi(\psi')}(^3P_0) > - < \mathcal{O}_8^{J/\psi(\psi')}(^1S_0) >]]$$

The Corresponding Production Cross Section Unpolarized pp Collisions is:

$$\sigma_{(pp\to J/\psi(\lambda)(\psi'(\lambda)))} = \frac{\pi^3\alpha_s^2}{27sm^3}\int_{4m^2/s}^{1}\frac{dx_1}{x_1}[f_q(x_1,2m)f_{\bar{q}}(\frac{4m^2}{x_1 s},2m)(1-\delta_{\lambda 0}) < \mathcal{O}_8^{J/\psi(\psi')}(^3S_1) > +\frac{15}{32}f_g(x_1,2m)f_g(\frac{4m^2}{x_1 s},2m)\times[\frac{9}{m^2}(1-\frac{2}{3}\delta_{\lambda 0}) < \mathcal{O}_8^{J/\psi(\psi')}(^3P_0) > + < \mathcal{O}_8^{J/\psi(\psi')}(^1S_0) >]]$$

The Spin Assymetry is Given By:

$A_{LL} = \frac{d\Delta\sigma}{d\sigma}$

The Following Non-Perturbative Matrix Elements are Extracted From The Tevatron data:

$\frac{4}{3} < \mathcal{O}_8^{J/\psi}(^1S_0) >= 0.022\mathrm{GeV}^3$

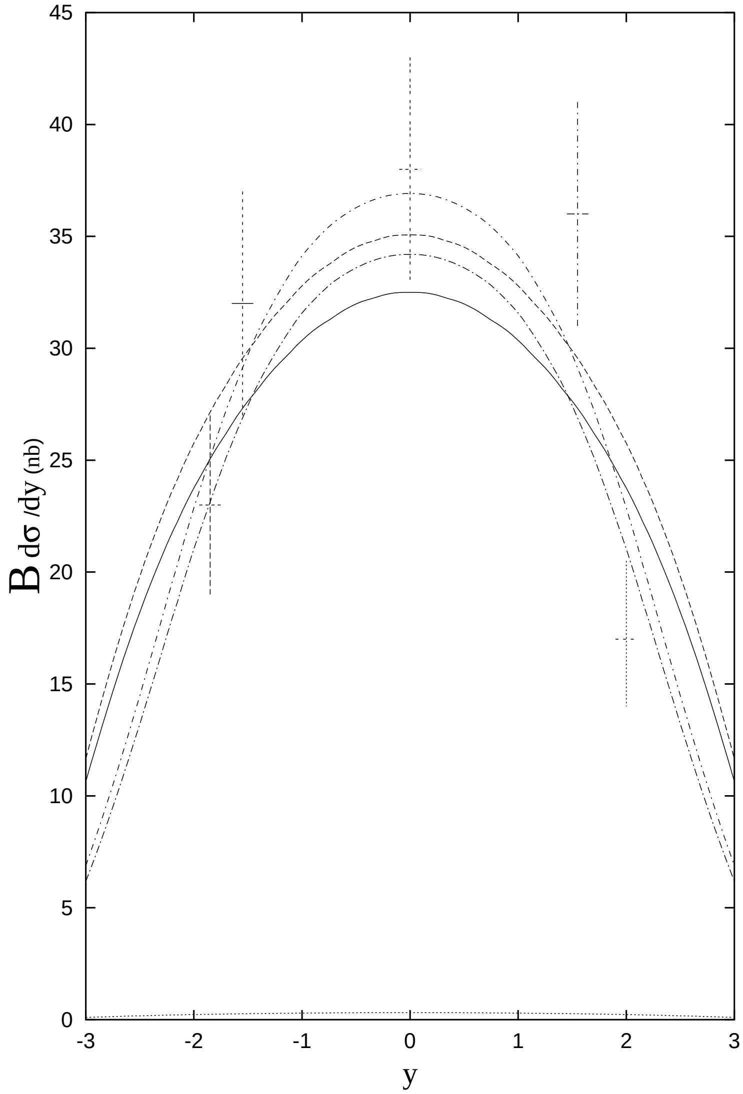
$< \mathcal{O}_8^{J/\psi}(^3S_1) >= 0.0066\mathrm{GeV}^3$

$\frac{4}{3m^2} \;\; < \mathcal{O}_8^{J/\psi}(^3P_0) >= 0.022\mathrm{GeV}^3$

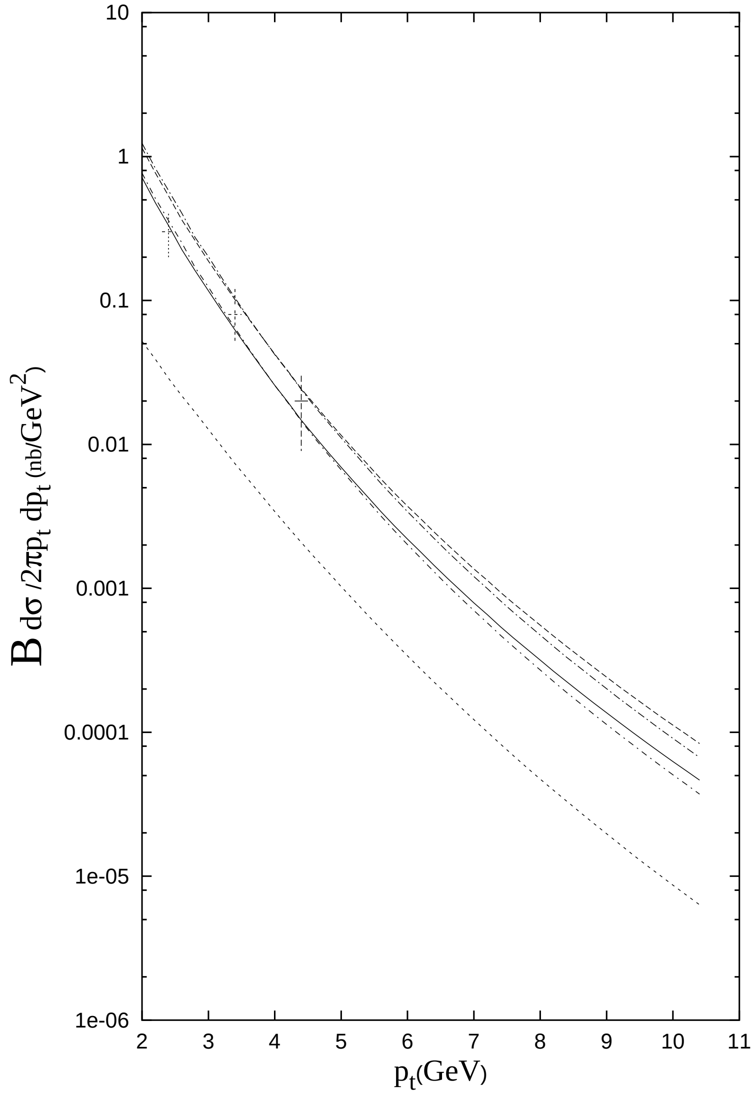
$\frac{4}{3} < \mathcal{O}_8^{\psi'}(^1S_0) >= 0.0059\mathrm{GeV}^3$

$< \mathcal{O}_8^{\psi'}(^3S_1) >= 0.0046\mathrm{GeV}^3$

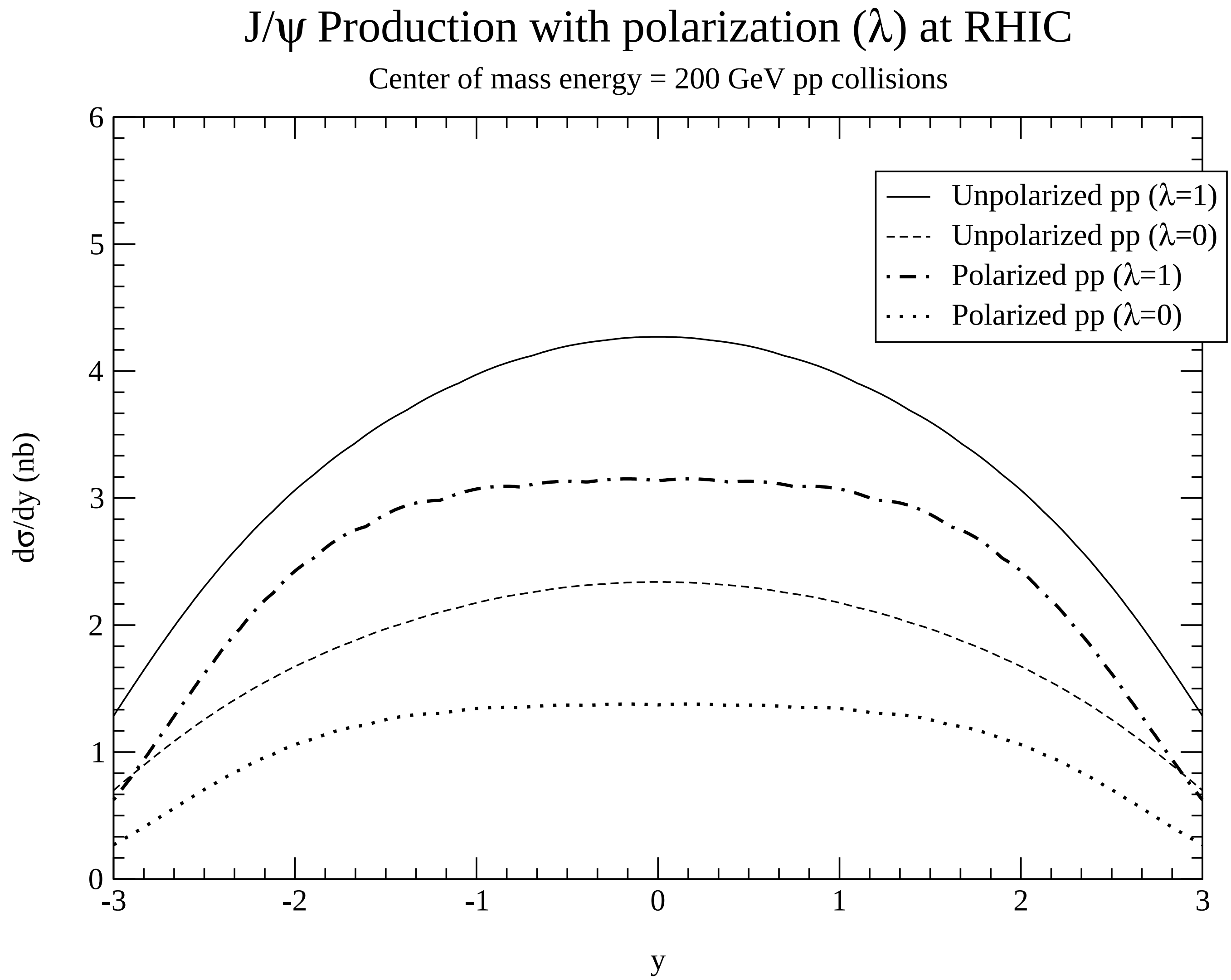
$\frac{4}{3m^2} < \mathcal{O}_8^{\psi'}(^3P_0) >= 0.0059\mathrm{GeV}^3$



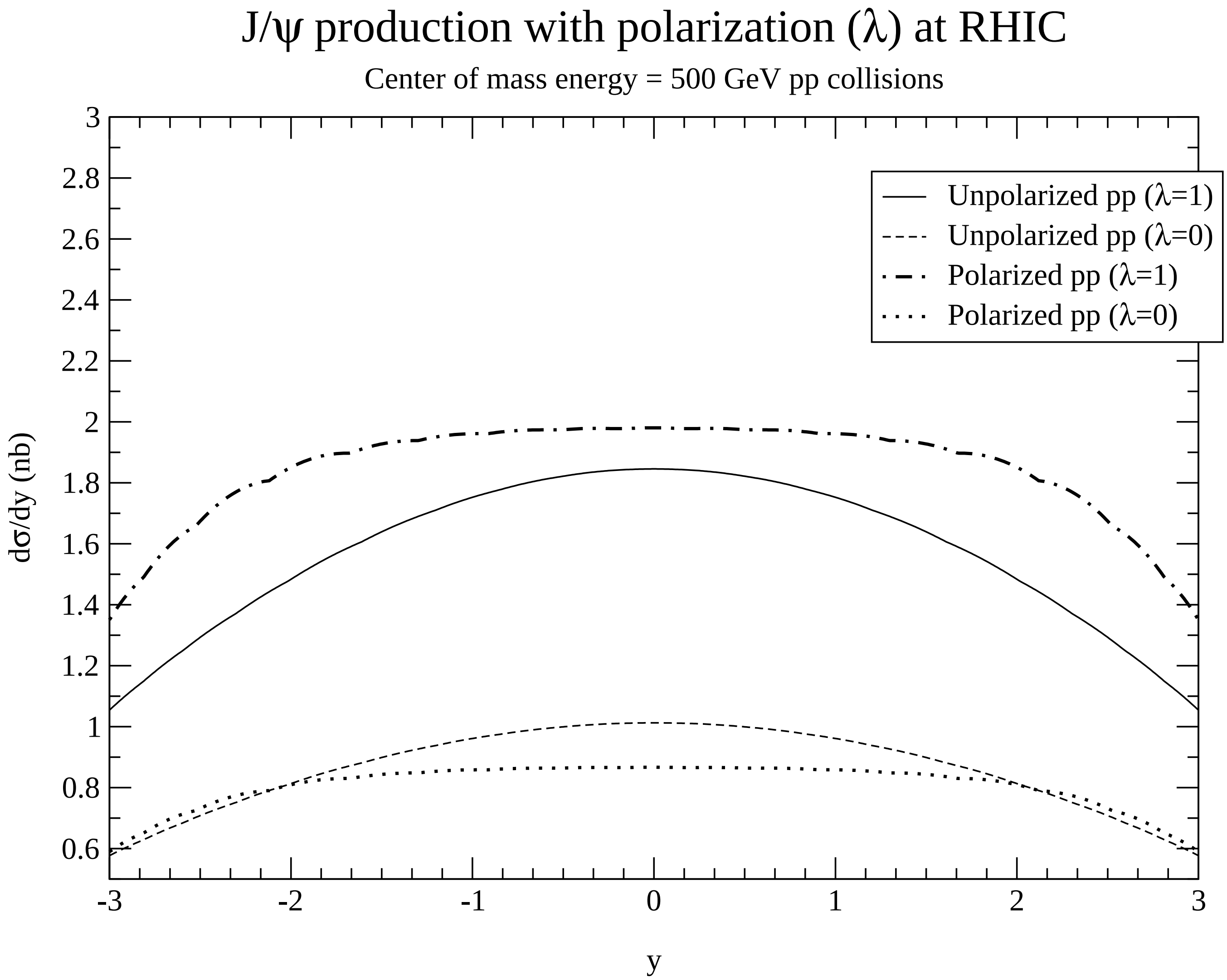
Unpolarized J/Ψ in Unpolarized pp Collisions at $\sqrt{s} = 200$ GeV at RHIC (F. Cooper, M. X. Liu and G. C. Nayak, Phys. Rev. Lett.93 (2004) 171801).



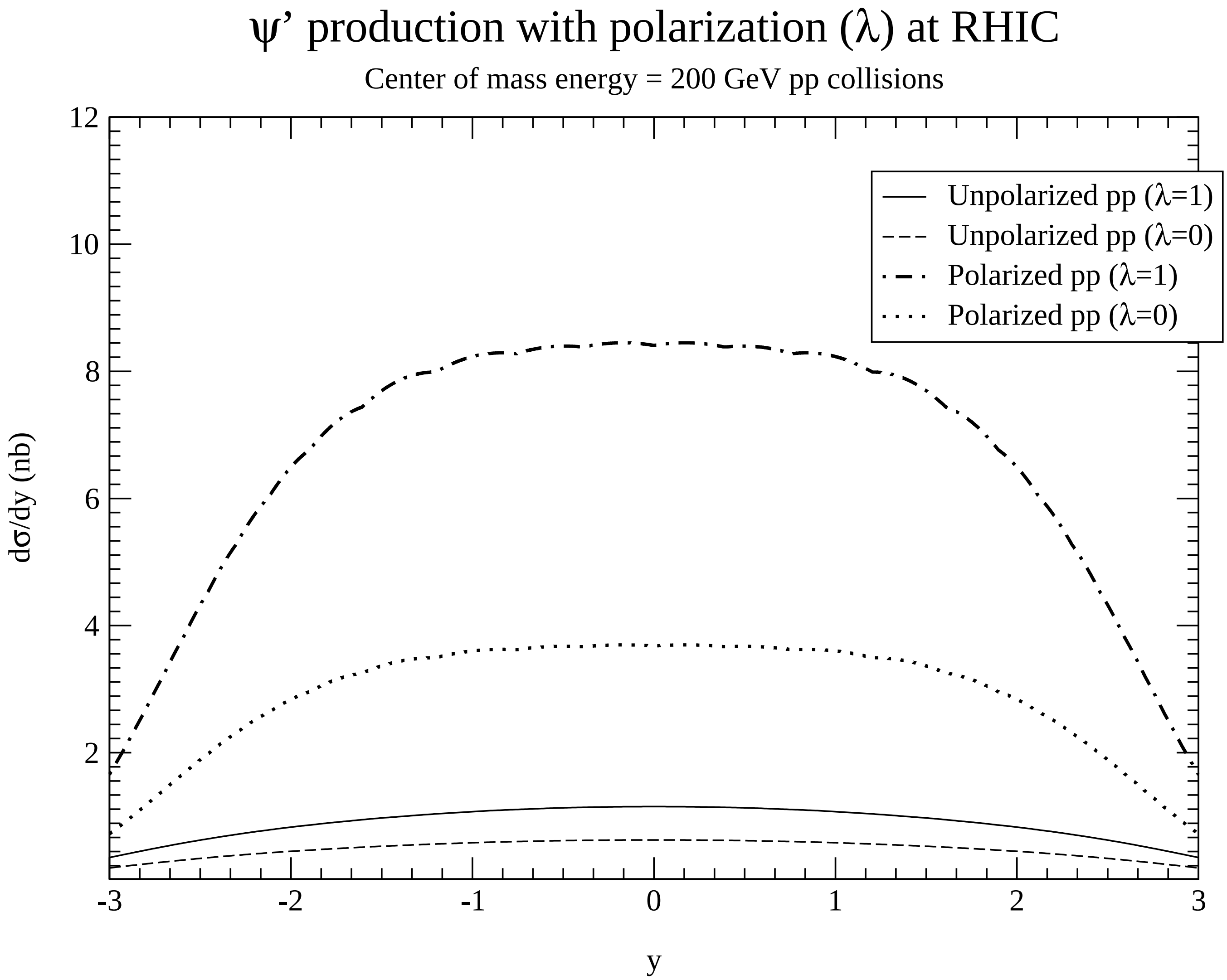
Unpolarized J/Ψ in Unpolarized pp Collisions at $\sqrt{s} = 200$ GeV at RHIC (F. Cooper, M. X. Liu and G. C. Nayak, Phys. Rev. Lett.93 (2004) 171801).



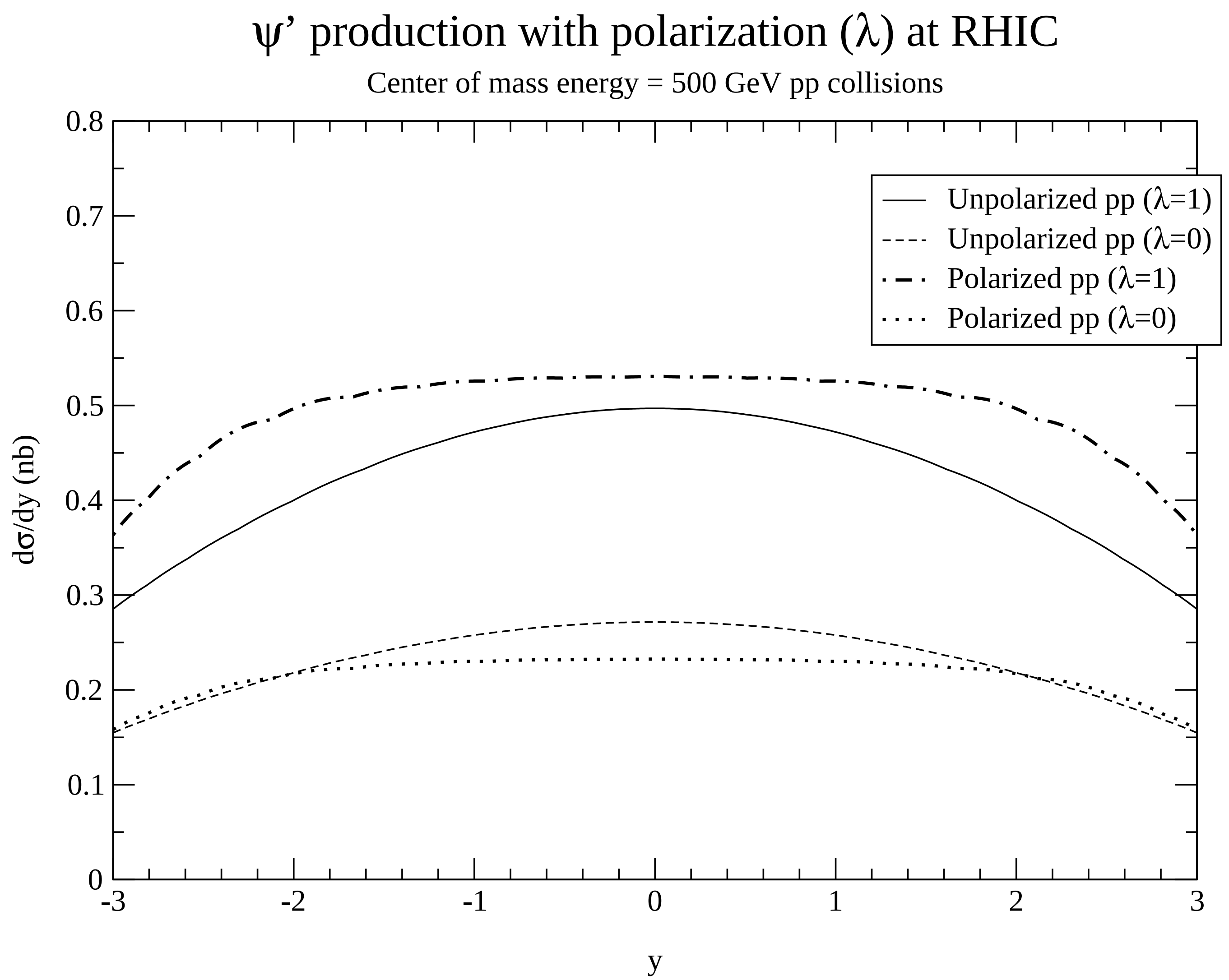
J/Ψ Polarization in Polarized pp Collisions at $\sqrt{s} = 200$ GeV at RHIC



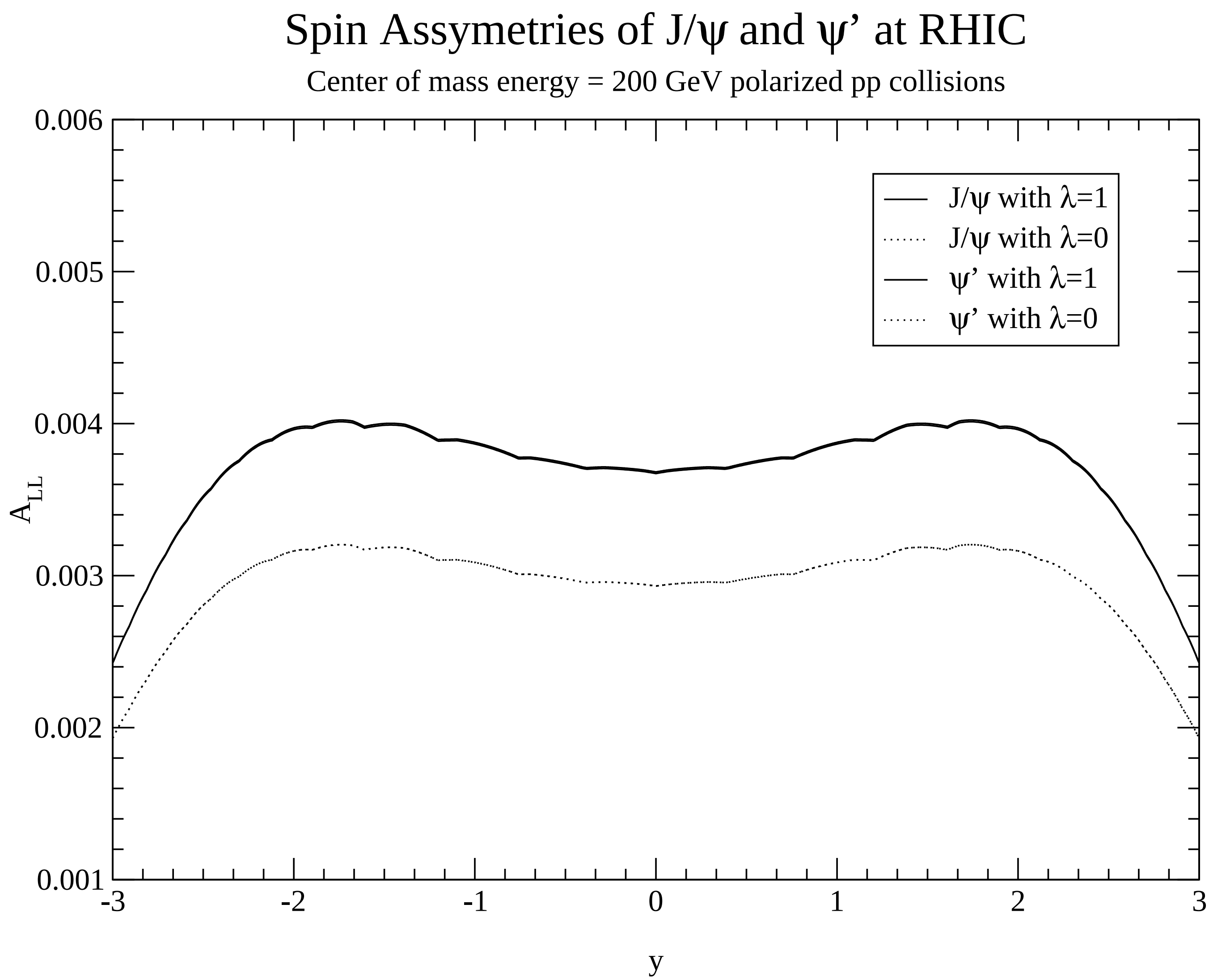
J/Ψ Polarization in Polarized pp Collisions at $\sqrt{s} = 500$ GeV at RHIC



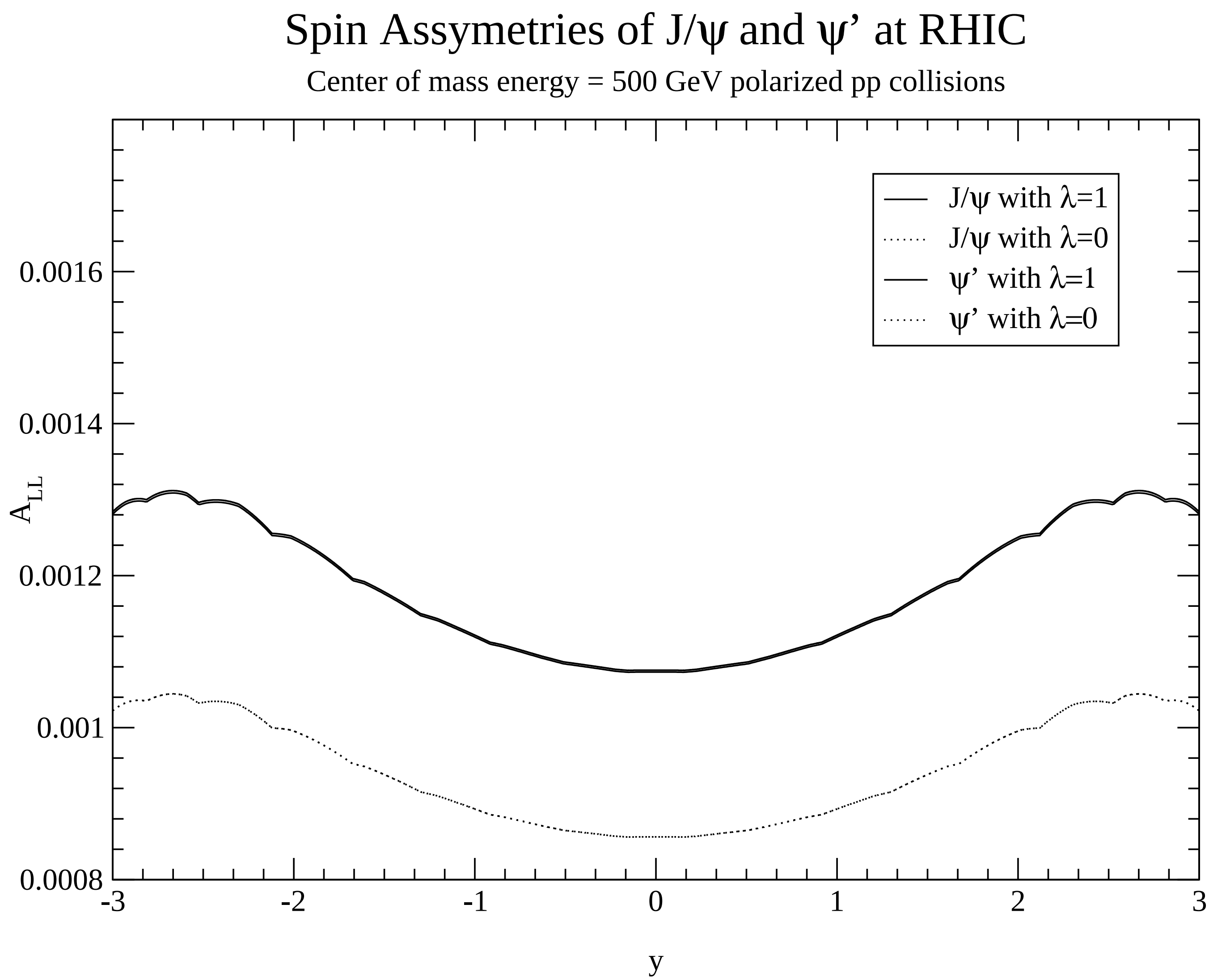
Ψ' Polarization in Polarized pp Collisions at $\sqrt{s} = 200$ GeV at RHIC



Ψ' Polarization in Polarized pp Collisions at $\sqrt{s} = 500$ GeV at RHIC



J/Ψ and Ψ' Spin Assymetry in Polarized pp Collisions at $\sqrt{s} = 200$ GeV at RHIC



J/Ψ and Ψ' Spin Assymetry in Polarized pp Collisions at $\sqrt{s} = 500$ GeV at RHIC

CONCLUSIONS:

We have Studied J/Ψ and Ψ' Polarizations in Polarized Proton-Proton Collisions at RHIC at $\sqrt{s} = 200$ GeV and 500 GeV.

Polarized Gluon Distribution Function Inside the Proton Can be Extracted From J/Ψ Measurement at RHIC.

As J/Ψ Polarization at Tevatron Energy is Not Explained by The Theory it is Useful to See What Happens at RHIC.

The J/Ψ Polarization Study in Polarized pp Collisions is Unique (Not Available at Any Other Collider) and It Tests the Spin Transfer Process in pQCD.